**Stony Brook University**
Mathematics Department
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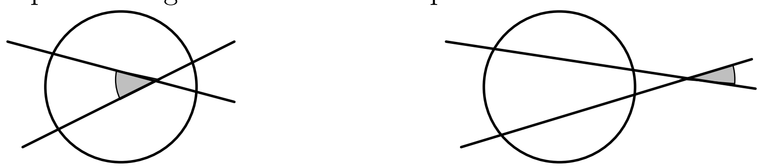
**Geometric Structures**
MAT 360
May 11, 2010

# Final Exam

**Problem 1. (8 pt)** Formulate and prove theorems about relationships between the angles formed by two intersecting lines and the arcs which are cut by the lines on a circle, which is not tangent to the lines and does not pass through their intersection point.



**Problem 2. (5 pt)** Construct a triangle $\triangle ABC$ given an angle congruent to its interior angle at vertex $A$, a segment congruent to a radius of inscribed circle, and a segment congruent to the altitude dropped from vertex $B$.

**Problem 3. (5 pt)** Given a convex quadrilateral $PQRS$ and a point $O$ inside of $PQRS$, construct a parallelogram $ABCD$ such that $A \in PQ$, $B \in QR$, $C \in RS$ and $D \in SP$ and $O$ is the intersection point of diagonals $AC$ and $BD$.

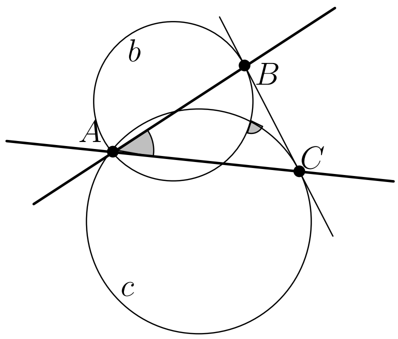
**Problem 4. (9 pt)** Find the interior angles of a triangle $\triangle ABC$, in which median $AM$ and altitude $AH$ divide angle $\angle A$ into three equal angles (i.e., $\angle CAH = \angle HAM = \angle MAB$).

**Problem 5. (8 pt)** Parallelepiped is a polyhedron bounded in the 3-space by three pairs of parallel planes.

(1) Prove that each face of a parallelepiped is a parallelogram.
(2) Formulate properties of a parallelepiped similar to the properties of a parallelogram that were studied in the course and prove one of them.

**Problem 6. (8 pt)** Prove that the image of a circle under an inversion is either a circle or a line.

**Problem 7. (12 pt)** On sides of a fixed angle with vertex $A$, one chooses points $B$ and $C$ and draws circles $b$ and $c$ passing through $A$ and tangent to $BC$ at points $B$ and $C$, respectively.




(1) Draw the image of this picture under an inversion centered at $A$.
(2) Prove that the angle between circles $b$ and $c$ that is marked on the picture above does not depend on the choice of $B$ and $C$.
(3) Find the relation of the angle between $b$ and $c$ to $\angle A$.

**Problem 8. (5 pt)** It is known that if a convex hexagon $ABCDEF$ can be inscribed in a circle, then the sum of interior angles at $A$, $C$ and $E$ is $360°$.

Show that the converse is not true. Even more, prove that it is impossible to recognize whether a convex hexagon can be inscribed in a circle if only interior angles are known.