Stony Brook University
Mathematics Department
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Foundations of Analysis
MAT 319
October 24, 2015

Program for Midterm 2.

The topics listed here are recommended for review. In the exam, it may be required to state the relevant definitions and theorems or use them in solutions of problems.

(1) Series, covergence and divergence of series, the sum (Section 17.1).
(2) Geometric series, condition of convergence, the sum (Example 1 in 17.1).
(3) Cauchy criterion for convergence (14.3-14.5).
(4) Comparison test (14.6).
(5) Absolute covergence (14.2 and 14.7).
(6) Ratio test (14.8) and root test (14.9) with proofs.
(7) Harmonic series and its divergence (15 Example 1).
(8) Convergence of $\sum \frac{1}{n^p}$ (15.1).
(9) Integral tests (15.2)
(10) Alternating series theorem 15.3
(11) The Riemann rearrangement theorem. A series which converges, but does not converge absolutely is called *conditionally convergent*. The Riemann rearrangement theorem states that for any real number M terms of a conditionally convergent series can be permutted so that the sum would become M. See Definitions, Statement of the theorem and Proof sections in the wikipaedia article

`https://en.wikipedia.org/wiki/Riemann_series_theorem`

(12) Invariance of the sum of a positive convergent series under permutations of its terms.
(13) Definition of a topological space. Open sets, topological structure in a set. Section 2.1 of the Complements.
(14) Neighborhoods of a point in a topological space. Section 2.1 of the Complements.
(15) Interior, exterior and boundary points of a set in a topological space. Section 2.1 of the Complements.
(16) Definitions of metric and metric spaces. Balls and spheres in a metric space. Section 2.4 of the Complements and Definition 13.1 in the textbook.
(17) Metric topology. Theorem 2.1 from the Complements, 13.6, 13.7 and 13.8 in the textbook.
(18) Topology of a subspace. Section 2.5 from the Complements.
(19) Definition of continuous maps between topological spaces and their simplest properties. Section 3.1 from the Complements.
(20) Continuity at a point and its relation to continuity. Section 3.2 from the Complements.
(21) Sequential continuity and its relation to Continuity. Section 3.3 from the Complements and Theorem 17.2 from the textbook.
(22) Theorems about operations with continuous functions. Theorems 17.3 and 17.4 from the textbook.
(23) Theorem on sequential continuity of composition of sequentually continuous maps. Theorem 17.5 from the textbook.
(24) Extremal Value Theorem (Theorem 18.1 from the textbook).
(25) Intermediate Value Theorem (Theorem 18.2 from the textbook).
(26) Theorems about continuity of monotone functions. (Theorems 18.4-18.6 from the textbook.)