MAT 310, Linear Algebra Name ____________________

Homework 4

Score____________

1. Let V and W be finite-dimensional vector spaces and let $S : V \to W$ and $T : W \to V$ be linear maps such that $TS = \mathrm{id}_V$.
(a) Express the dimensions of the null spaces and ranges of S and T in terms of dimensions of V and W.
(b) Which values $\dim V$ and $\dim W$ can take in this situation?

2. Let U, V, W be finite-dimensional vector spaces.
(a) If any linear map $T : U \to W$ can be presented as a composition of linear maps $U \to V \to W$, then what can be the dimensions of the spaces?
(b) What linear maps $U \to W$ can be presented as compositions $U \to V \to W$?

3. Find a basis of the subspace $\{(x_1, x_2, x_3, x_4, x_5, x_6) \in \mathbb{F}^6 \mid x_2 = 4x_1 \text{ and } x_6 = x_3 = x_4\}$.

4. For which vector spaces V the set of non-invertible operators $V \to V$ is a subspace of $\mathcal{L}(V)$?

5. Let V be a finite-dimensional vector space, and $T_1, T_2, \ldots, T_n$ be linear maps $V \to V$. Prove or find counter-example: if the composition $T_1 T_2 \ldots T_n$ is surjective then each of T_i is surjective.