

Homework 2

1. Let $f : X \rightarrow Y$ and $g : Y \rightarrow X$ be continuous maps homotopy inverse to each other and M_f be the mapping cylinder of f . Let $H : X \times I \rightarrow X$ be a homotopy between $g \circ f$ and id_X and $G : Y \times I \rightarrow Y$ a homotopy between $f \circ g$ and id_Y . Construct a deformation retraction $M_f \rightarrow X$ and homotopy which proves that this is a deformation retraction.
2. Find a homotopy equivalence between S^1 and the space of quadratic polynomials $x^2 + px + q$ with complex p and q and no double root.
3. Are $S^2 \setminus \{a, b, c\}$ and $S^1 \setminus \{e, f\}$, where a, b, c, e, f are pairwise different points, homotopy equivalent?
- 4 **Riddle.** Use Gram-Schmidt orthogonalization process to deduce a theorem which claims existing of a deformation retraction.