# Lecture 4

# Sets

# A set and its elements



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one of the most profound actions is

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The **first words** in this language are **set** and **element**.

A **set** is a collection of objects which are called **elements**.
A set **consists** of (and is **defined** by) its elements.

# Notations and synonims

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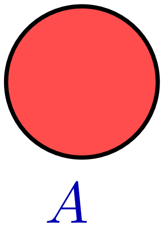

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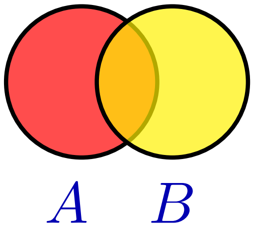

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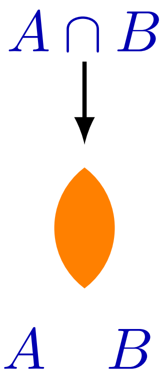

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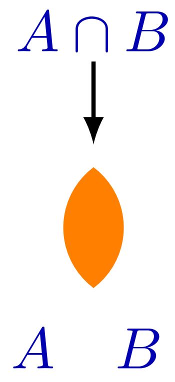


Venn diagram

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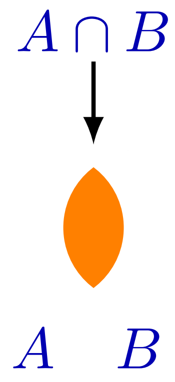


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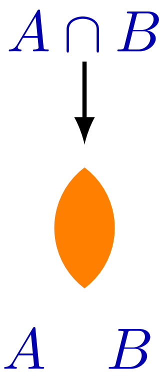


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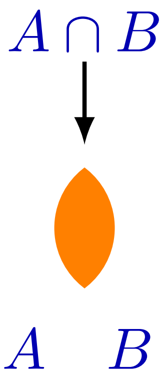


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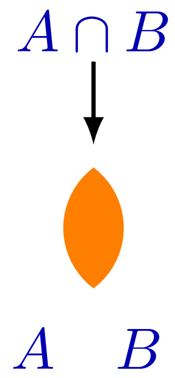


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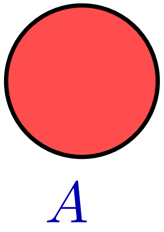

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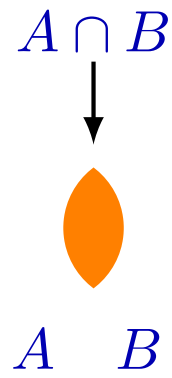


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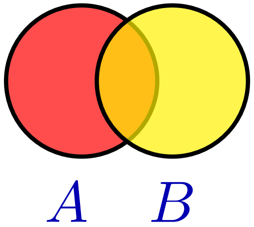

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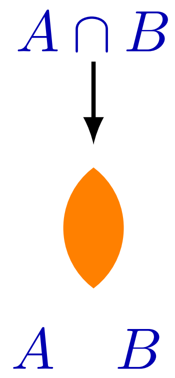


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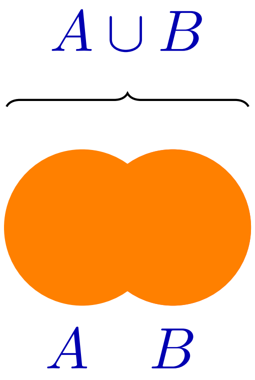

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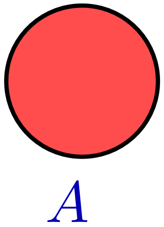

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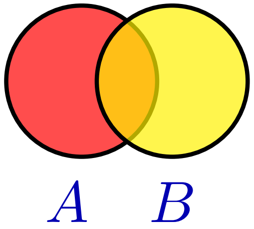

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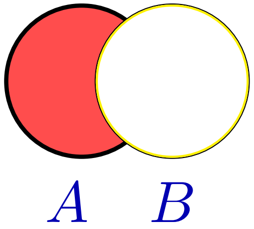

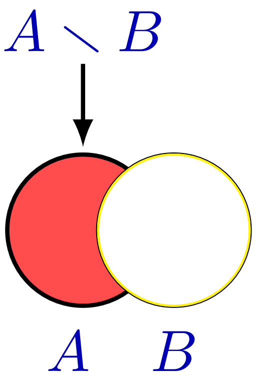


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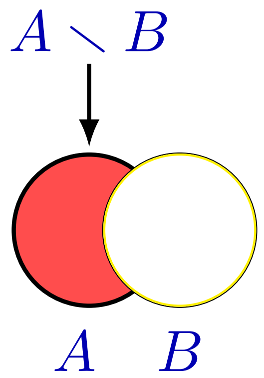


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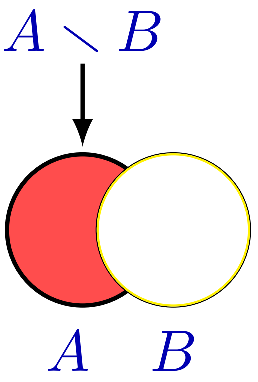


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$A^C$

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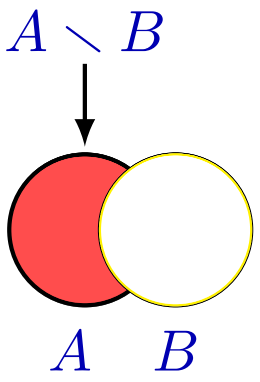


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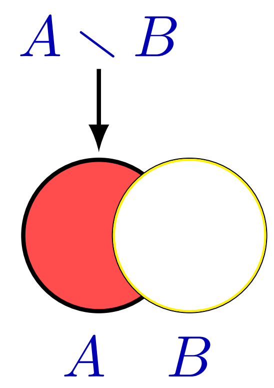


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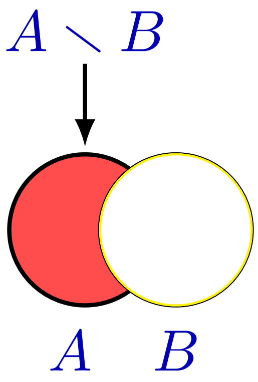


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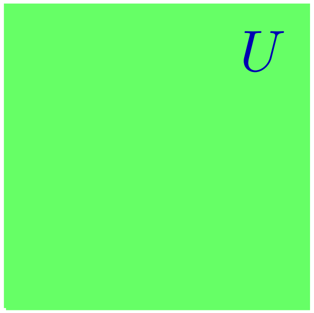

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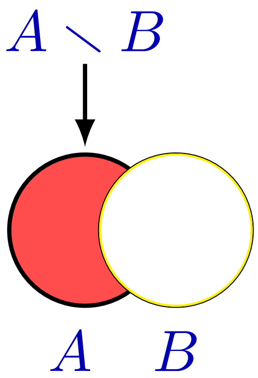


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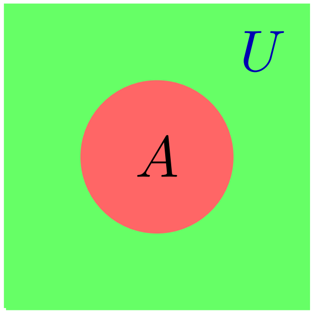

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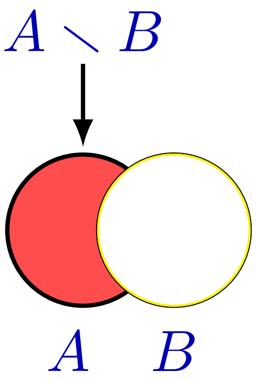


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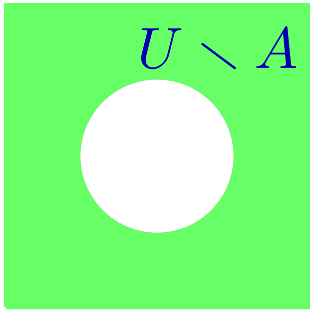

# Simplest set-theoretical identities

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**Definition.** Sets $A$ and $B$ are called **disjoint** if $A \cap B = \varnothing$.

## Set-builder notation



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This **set-builder notation** unveils a close relation between **predicates** and **sets**:

Every predicate $P(x)$ defines a subset $\{x \in A \mid P(x)\}$ of $A$.

Vice versa, every subset $B \subset A$ gives rise to a predicate $x \in B$.

# Logic vs. set theory

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|---|---|---|---|---|---|---|---|---|
| | | | | | | | | |

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$(A \cap B)^c = A^c \cup B^c$ and $(A \cup B)^c = A^c \cap B^c$.

# Proving set-theoretic identities: De Morgan's law

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**Example 1.**

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Therefore, $(A \cap B)^c \subset A^c \cup B^c \quad (*)$.

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Prove now that $A^c \cup B^c \subset (A \cap B)^c$ .

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Therefore, $(A \cap B)^c \subset A^c \cup B^c \quad (*)$ .

Prove now that $A^c \cup B^c \subset (A \cap B)^c$ .

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Combining $(*)$ and $(**)$, we get $(A \cap B)^c = A^c \cup B^c$. □

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# How to prove set-theoretic identities

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**Example 2.**

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**Illustration**

## How to prove set-theoretic identities

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**Illustration** (not a proof!):

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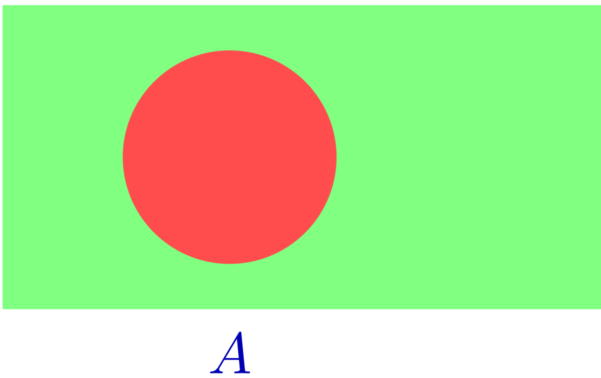

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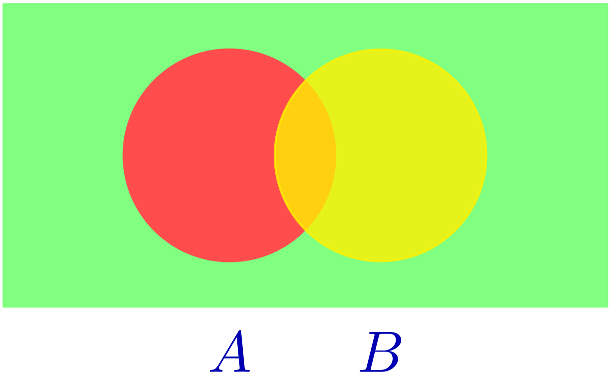

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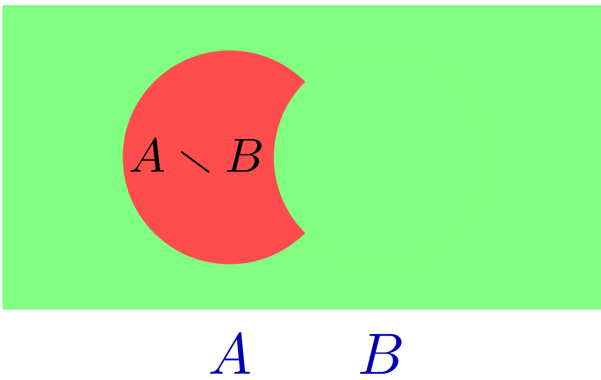

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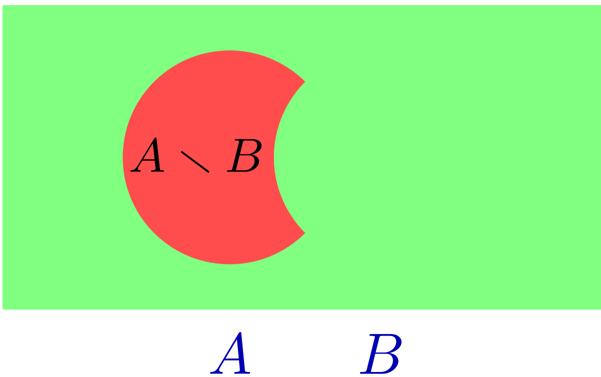

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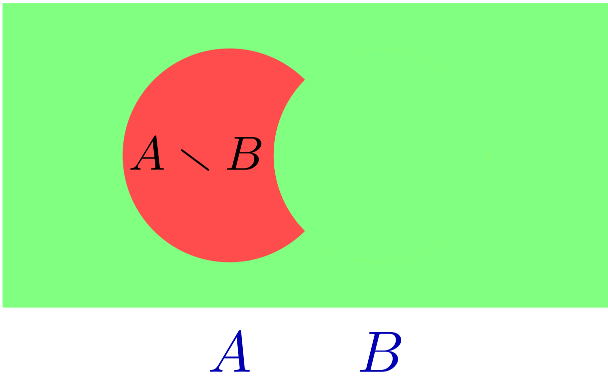


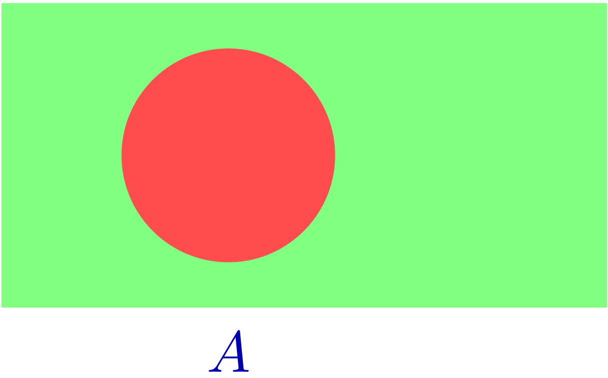

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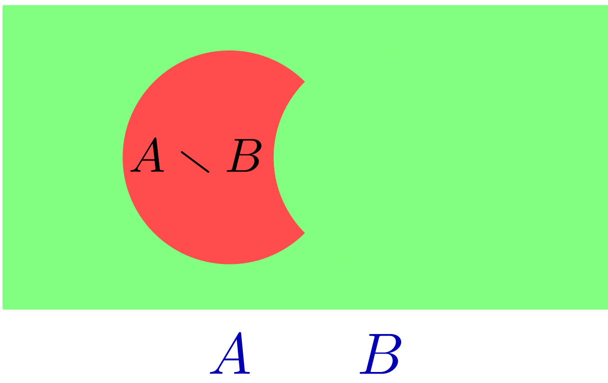


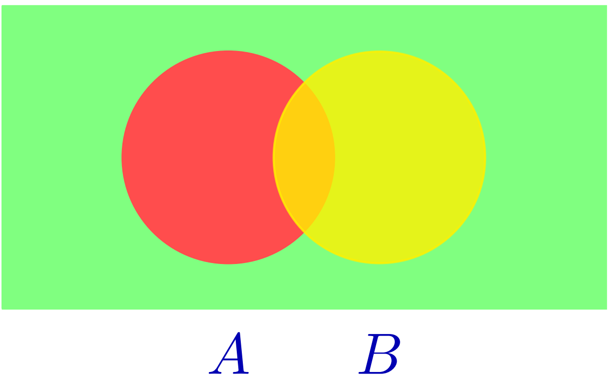

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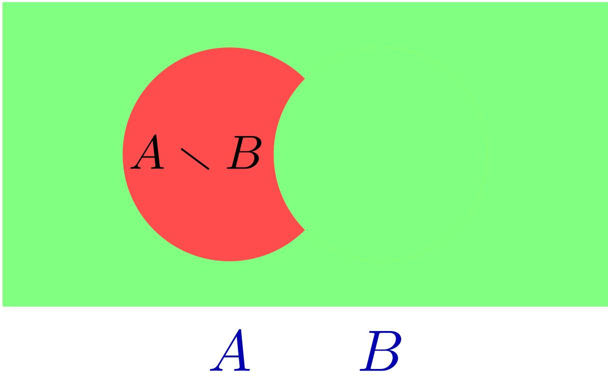


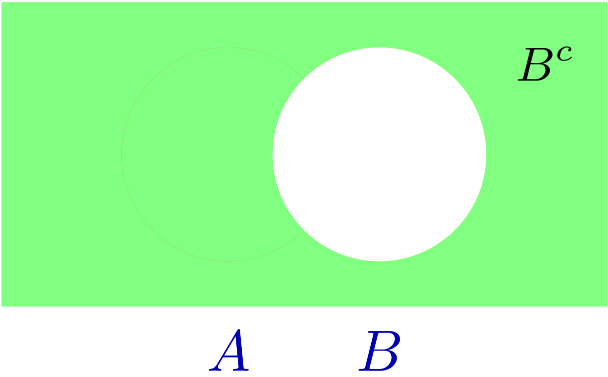

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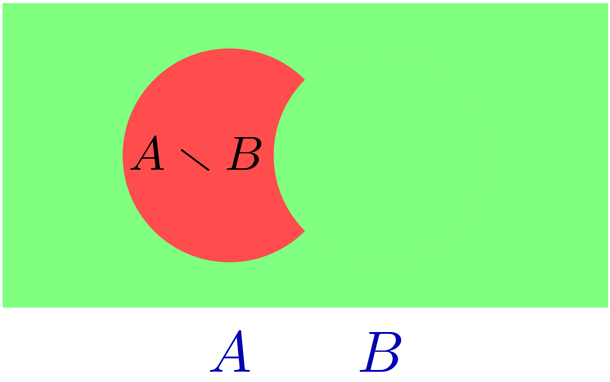


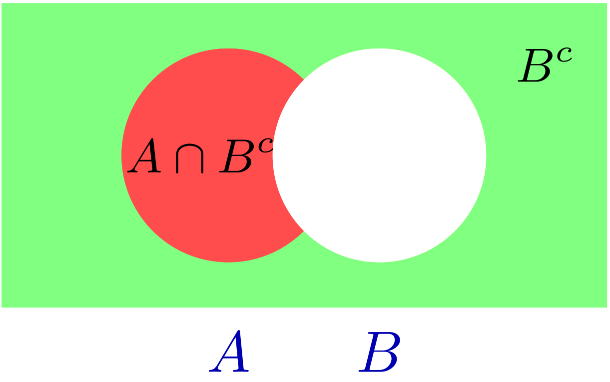

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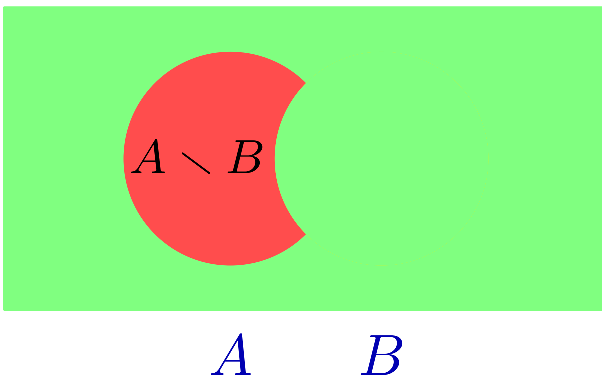


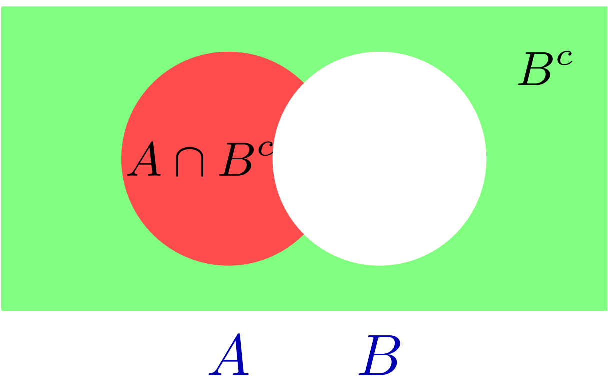


**Proof.**

# How to prove set-theoretic identities

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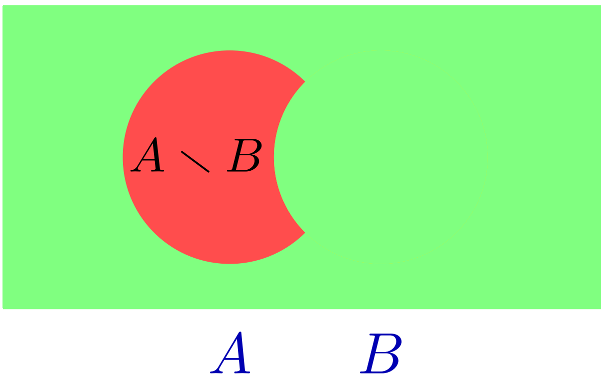


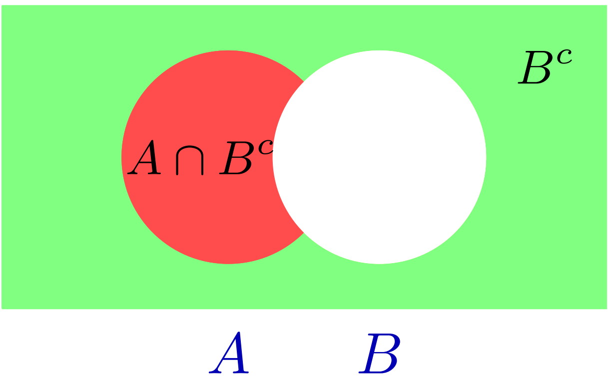


**Proof.** Alternative 1

# How to prove set-theoretic identities

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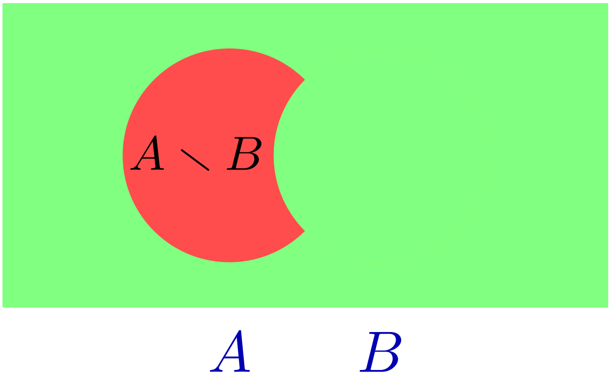
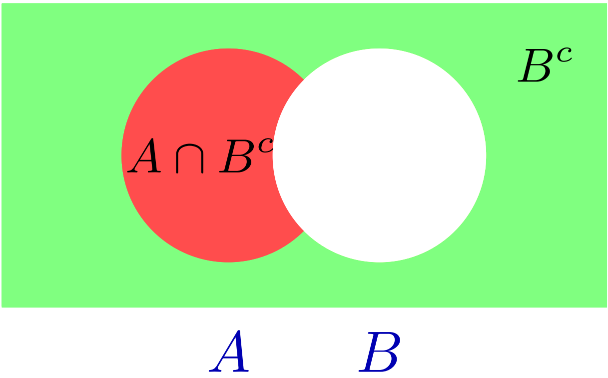


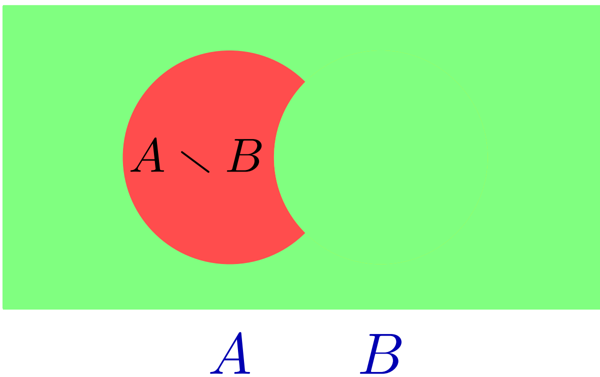



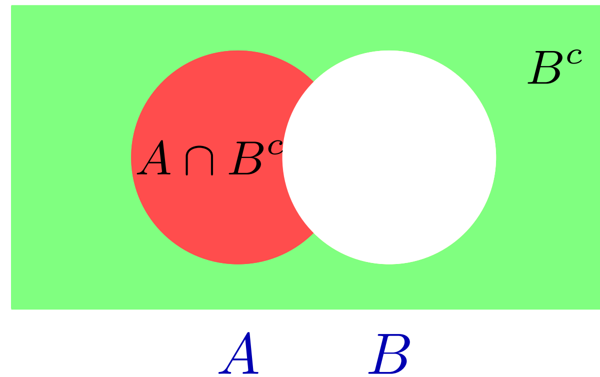

**Proof.** Alternative 1 (element-wise)

# How to prove set-theoretic identities

**Example 2.** Prove that $A \setminus B = A \cap B^c$ for any sets $A, B$.

**Illustration** (not a proof!):




**Proof.** Alternative 1 (element-wise)

$A \setminus B =$

# How to prove set-theoretic identities



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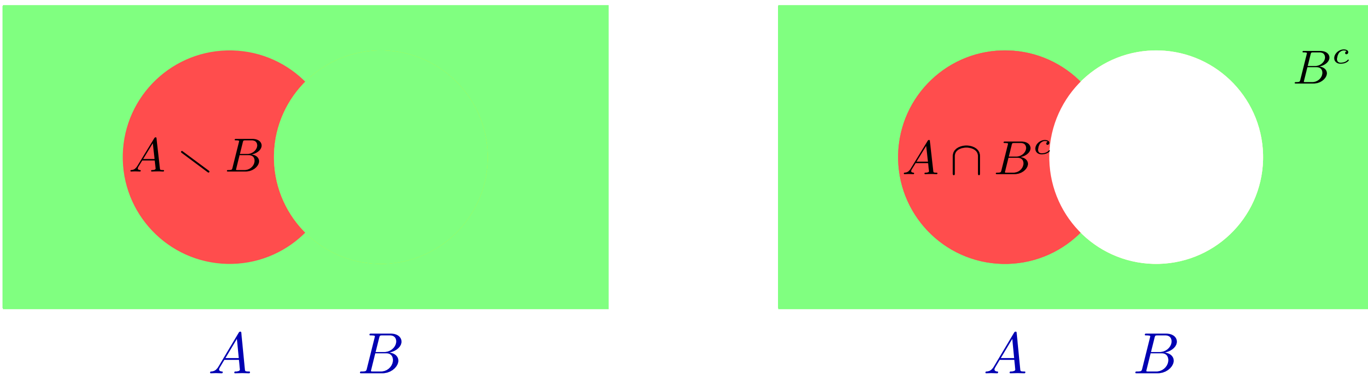


**Proof.** Alternative 1 (element-wise)

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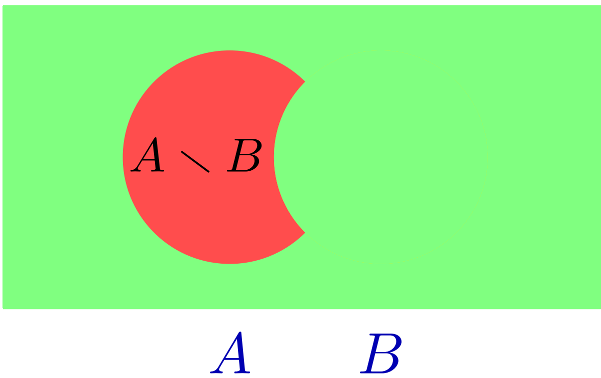


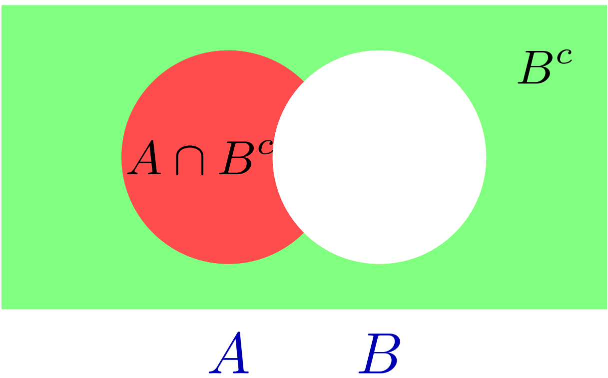


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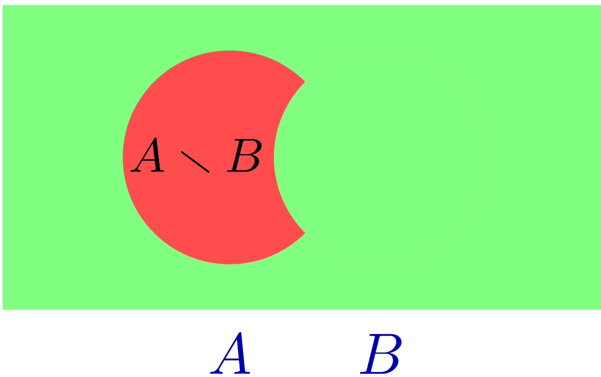


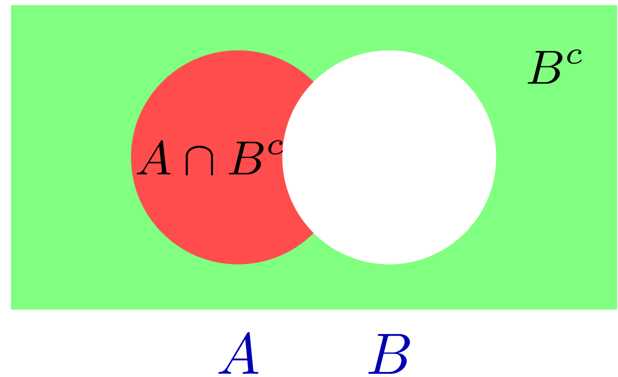


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$$A \setminus B = \{x \mid x \in A \wedge x \notin B\} = \{x \mid x \in A \wedge x \in B^c\} = A \cap B^c$$

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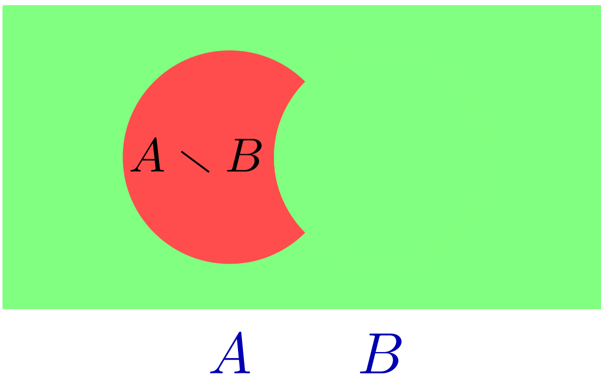


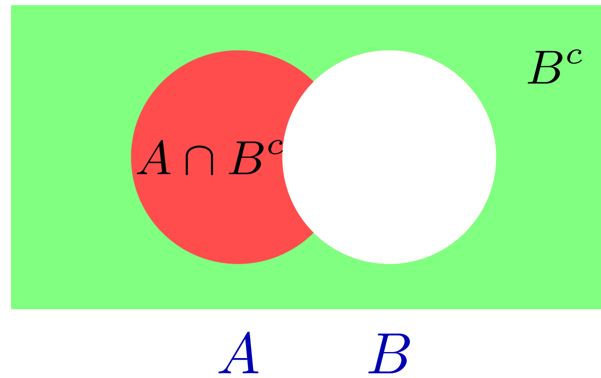


**Proof.** Alternative 1 (element-wise)

$$A \setminus B = \{x \mid x \in A \wedge x \notin B\} = \{x \mid x \in A \wedge x \in B^c\} = A \cap B^c \qquad \square$$

# How to prove set-theoretic identities

Alternative 2

# How to prove set-theoretic identities

Alternative 2 (two sets are equal iff each of them is a subset of the other)

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$$A \setminus B \subset U \setminus B = B^c$$

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We have got that $A \setminus B \subset A \cap B^c$

Alternative 2 (two sets are equal iff each of them is a subset of the other)

To prove $A \setminus B = A \cap B^c$, we prove that

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We have got that $A \setminus B \subset A \cap B^c$ and $A \setminus B \supseteq A \cap B^c$.

# How to prove set-theoretic identities

Alternative 2 (two sets are equal iff each of them is a subset of the other)

To prove $A \setminus B = A \cap B^c$, we prove that

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We have got that $A \setminus B \subset A \cap B^c$ and $A \setminus B \supseteq A \cap B^c$.

Therefore, $A \setminus B = A \cap B^c$. $\square$

# How to prove set-theoretic identities



Alternative 3

# How to prove set-theoretic identities

Alternative 3 (by truth table)

# How to prove set-theoretic identities

Alternative 3 (by truth table)

| | $x \in A$ | $x \in B$ | $x \notin B$ | $\underbrace{x \in A \wedge x \notin B}_{x \in A \setminus B}$ | $\underbrace{x \in A \wedge x \in B^c}_{x \in A \cap B^c}$ |
|---|---|---|---|---|---|
| 1 | T | T | F | F | F |
| 2 | T | F | T | T | T |
| 3 | F | T | F | F | F |
| 4 | F | F | T | F | F |

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| 1 | T | T | F | F | F |
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Since the last two columns of the truth table are identical,

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|---|---|---|---|---|---|
| 1 | T | T | F | F | F |
| 2 | T | F | T | T | T |
| 3 | F | T | F | F | F |
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Since the last two columns of the truth table are identical, $A \smallsetminus B = A \cap B^c$.

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| 2 | T | F | T | T | T |
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Since the last two columns of the truth table are identical, $A \setminus B = A \cap B^c$.

**Remark.**

# How to prove set-theoretic identities

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Since the last two columns of the truth table are identical, $A \setminus B = A \cap B^c$.

**Remark.** The universe can be presented as a **disjoint union**

# How to prove set-theoretic identities

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| 1 | T | T | F | F | F |
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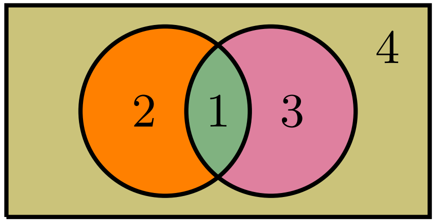
| 4 | F | F | T | F | F |

Since the last two columns of the truth table are identical, $A \setminus B = A \cap B^c$ .

**Remark.** The universe can be presented as a **disjoint union**

$$U = (A \cap B) \cup (A \setminus B) \cup (B \setminus A) \cup (A \cup B)^c$$



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**Remark.** The universe can be presented as a **disjoint union**

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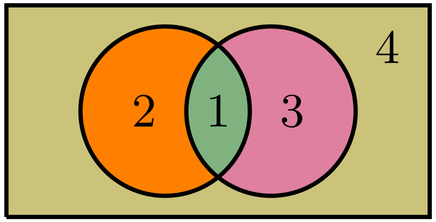


What does this formula remind you?

# How to prove set-theoretic identities

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| 2 | T | F | T | T | T |
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Since the last two columns of the truth table are identical, $A \smallsetminus B = A \cap B^c$.

**Remark.** The universe can be presented as a **disjoint union**

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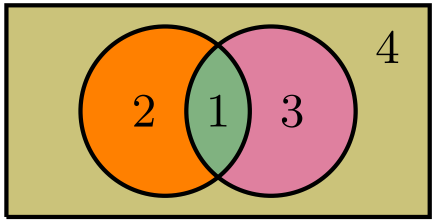


What does this formula remind you? Is it related to disjunctive normal form?

# How to prove set-theoretic identities

# How to prove set-theoretic identities

**Example 3.**

# How to prove set-theoretic identities



**Example 3.** Prove that $A \subset B \iff A \setminus B = \varnothing$

# How to prove set-theoretic identities



**Example 3.** Prove that $A \subset B \iff A \setminus B = \varnothing$ for any sets $A, B$.

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**Example 3.** Prove that $A \subset B \iff A \setminus B = \varnothing$ for any sets $A, B$.

**Proof.**

# How to prove set-theoretic identities

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**Proof.** Let $A \subset B$.

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## How to prove set-theoretic identities

**Example 3.** Prove that $A \subset B \iff A \setminus B = \varnothing$ for any sets $A, B$.

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$\forall x \in A$

# How to prove set-theoretic identities

**Example 3.** Prove that $A \subset B \iff A \setminus B = \varnothing$ for any sets $A, B$.

**Proof.** Let $A \subset B$. Then
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## How to prove set-theoretic identities



**Example 3.** Prove that $A \subset B \iff A \setminus B = \varnothing$ for any sets $A, B$.

**Proof.** Let $A \subset B$. Then
$\forall x \in A \quad x \in A \implies x \in B \implies x \notin B^c$.

# How to prove set-theoretic identities

**Example 3.** Prove that $A \subset B \iff A \setminus B = \varnothing$ for any sets $A, B$.

**Proof.** Let $A \subset B$. Then
$\forall x \in A \quad x \in A \implies x \in B \implies x \notin B^c$.
So any $x$ in $A$ doesn't belong to $B^c$.

# How to prove set-theoretic identities

**Example 3.** Prove that $A \subset B \iff A \setminus B = \varnothing$ for any sets $A, B$.

**Proof.** Let $A \subset B$. Then
$\forall x \in A \quad x \in A \implies x \in B \implies x \notin B^c$.
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Combining $(*)$ and $(**)$, we get $A \subset B \iff A \setminus B = \varnothing$. $\square$

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