MAT 220, Vector Geometry and Algebra

## Homework 3

Name $\qquad$

Score $\qquad$

1. Given lines $l_{1}$ and $l_{2}$, which intersect at a point $O$, and a point $A$, which does not belong to the lines. Is it possible to present the vector $\overrightarrow{A B}$ as the sum of two vectors, that are directed along $l_{1}$ and $l_{2}$ ? If so, then how to find these vectors?
2. Given a pentagon $A B C D E$, does there exit a pentagon such that its sides are parallel and congruent to the diagonals of $A B C D E$, that is segments $A C, A D, B D, B E, C E$ ?
3. Let $O$ be the center of regular hexagon $A B C D E F$. Express vectors $\overrightarrow{O A}, \overrightarrow{O B}, \overrightarrow{O C}$, and $\overrightarrow{O D}$ via $\mathbf{p}=\overrightarrow{O E}$ and $\mathbf{q}=\overrightarrow{O F}$.
4. Given a quadrilateral and a point $M$. Prove that the points symmetric to $M$ with respect to the midpoints of the sides of the quadrilateral are vertices of a parallelogram.
5. Given vectors $\mathbf{a}=\overrightarrow{O A}$ and $\mathbf{b}=\overrightarrow{O B}$, find a vector parallel to the bisector of the angle $\angle A O B$.

6 . Let $A_{1} A_{2} \ldots A_{2 n}$ be a regular $2 n$-gon. Prove that

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\overrightarrow{A_{1} A_{2}}+\overrightarrow{A_{1} A_{3}}+\cdots+\overrightarrow{A_{1} A_{2 n}}=n \cdot \overrightarrow{A_{1} A_{n+1}}
$$

