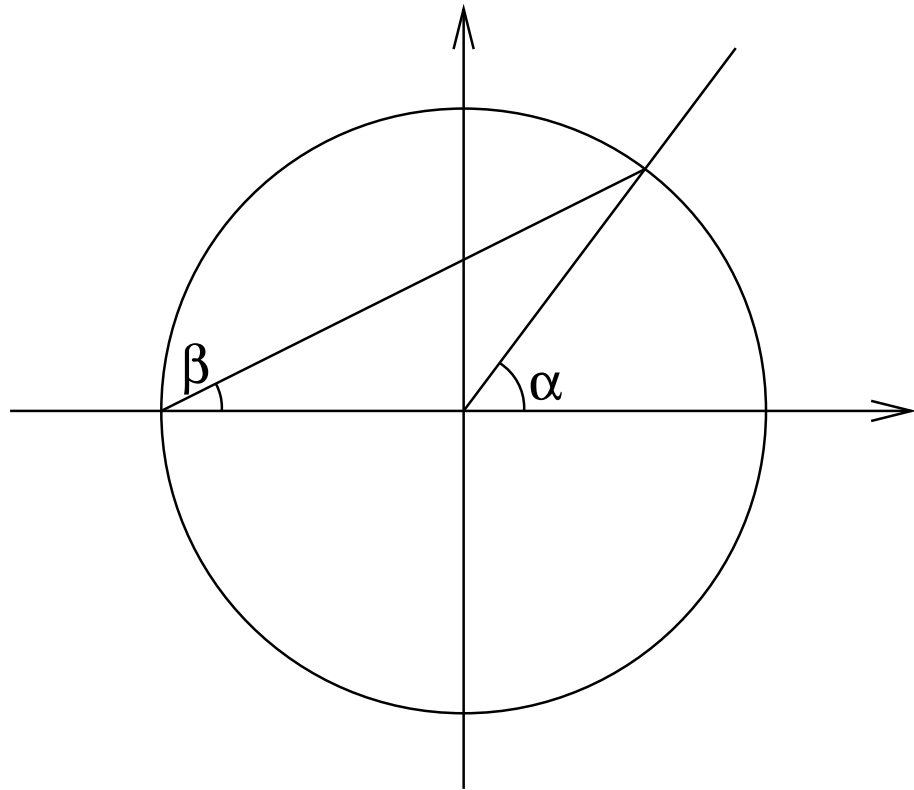

Homework 2

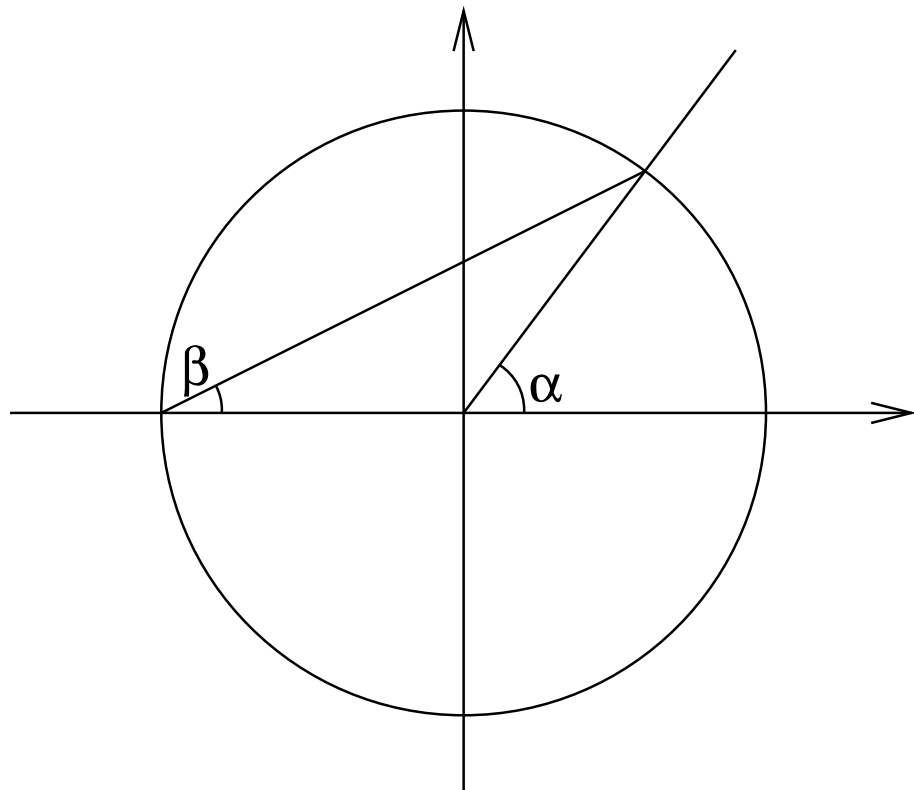
Oleg Viro

February 10, 2016

Stereographic projection

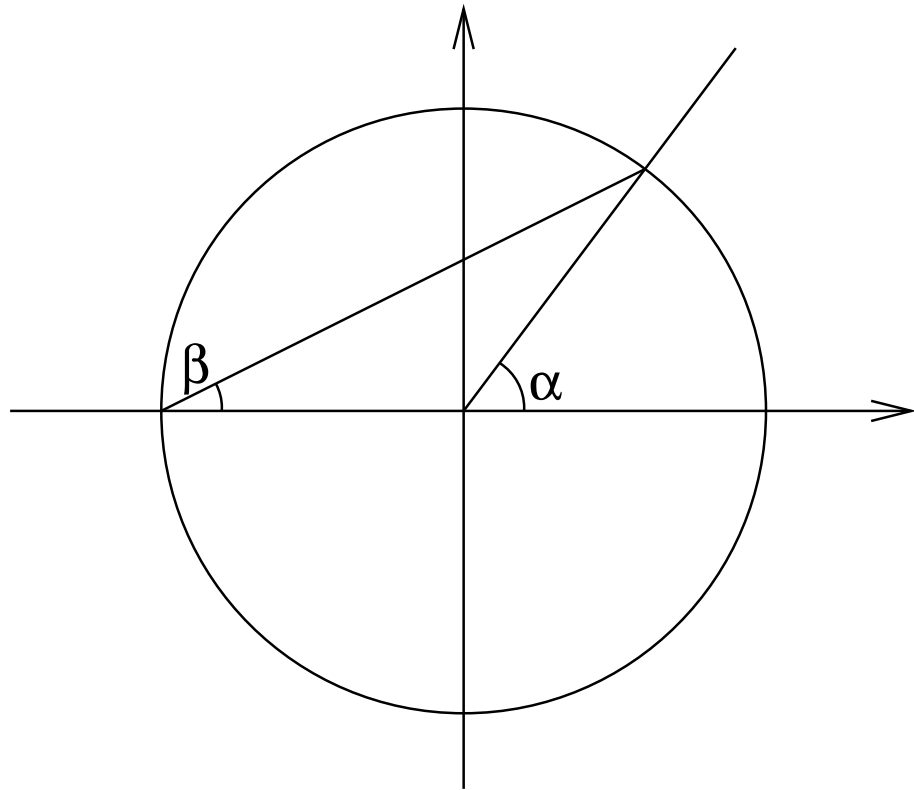


Stereographic projection



$(0, t) \mapsto (x, y)$, where (x, y) is the intersection point of $x^2 + y^2 = 1$ and $y = t(x + 1)$.

Stereographic projection

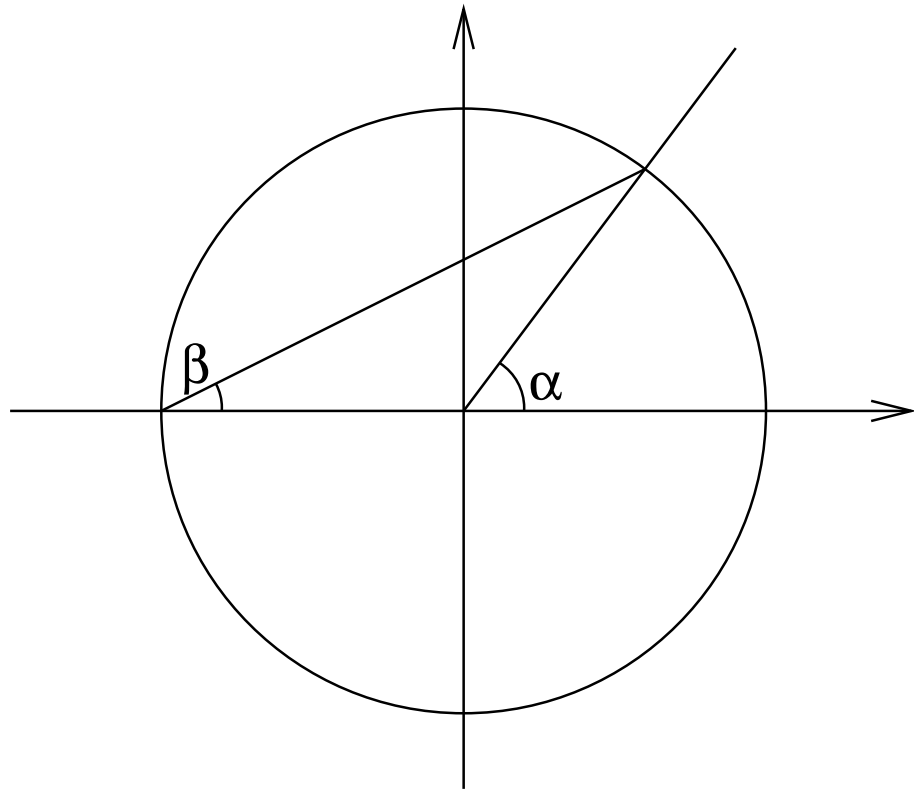


$(0, t) \mapsto (x, y)$, where (x, y) is the intersection point of $x^2 + y^2 = 1$ and $y = t(x + 1)$.

Substitute:

$$x^2 + t^2(x + 1)^2 = 1.$$

Stereographic projection



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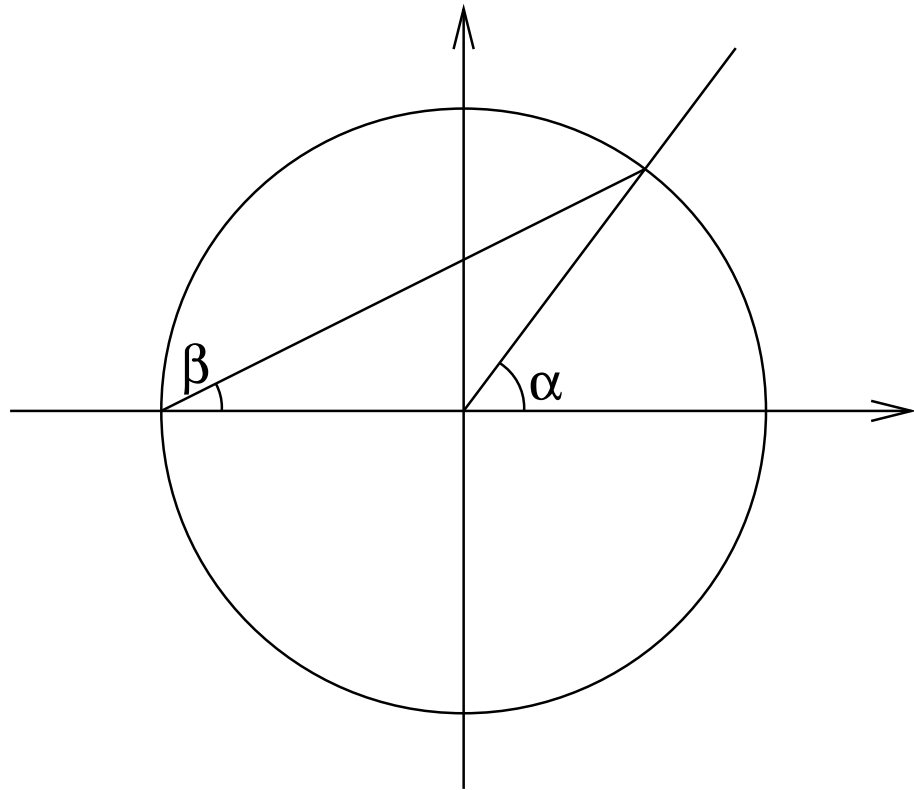
Substitute:

$$x^2 + t^2(x + 1)^2 = 1.$$

Simplify:

$$(1 + t^2)x^2 + 2t^2x + t^2 - 1 = 0$$

Stereographic projection



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Substitute:

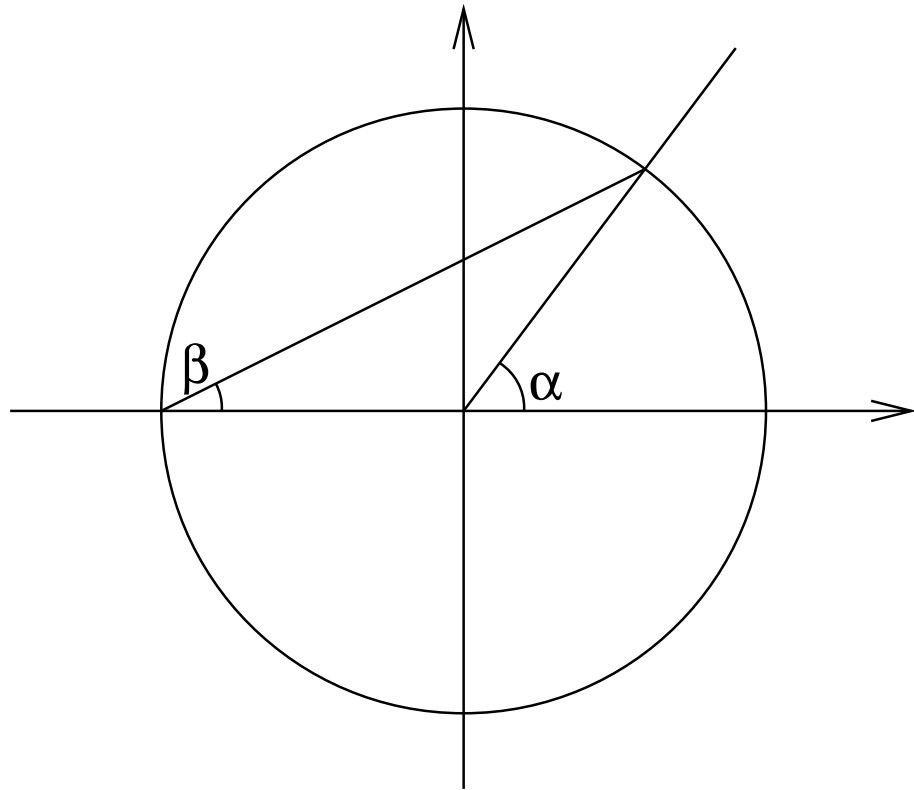
$$x^2 + t^2(x + 1)^2 = 1.$$

Simplify:

$$(1 + t^2)x^2 + 2t^2x + t^2 - 1 = 0$$

Divide by $x + 1$.

Stereographic projection



$(0, t) \mapsto (x, y)$, where (x, y) is the intersection point of $x^2 + y^2 = 1$ and $y = t(x + 1)$.

Substitute:

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Simplify:

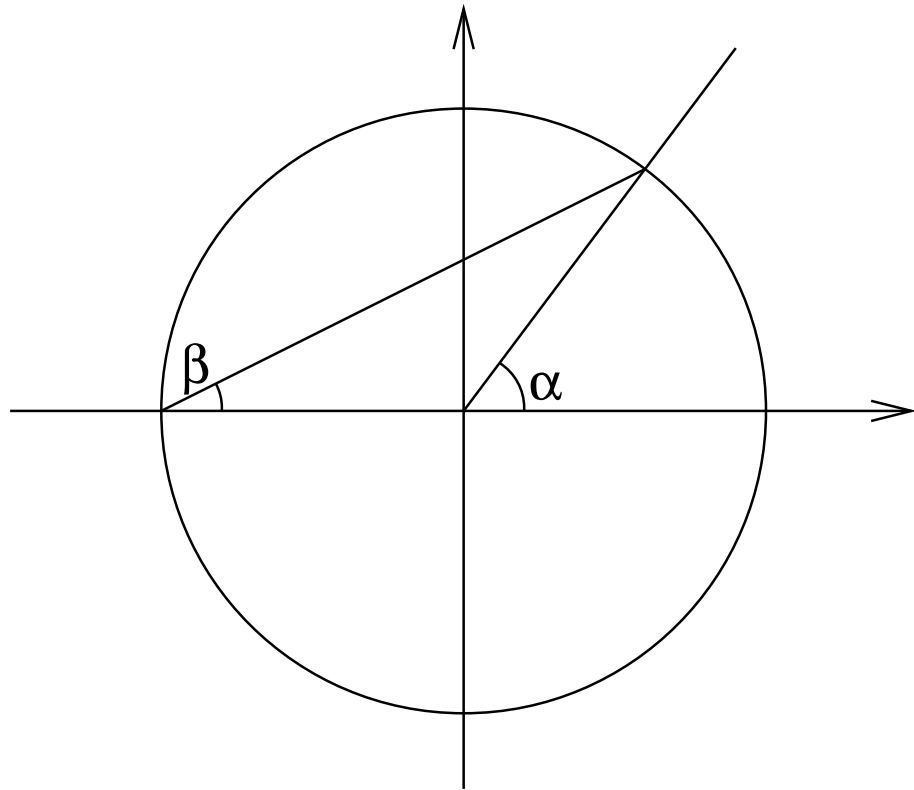
$$(1 + t^2)x^2 + 2t^2x + t^2 - 1 = 0$$

Divide by $x + 1$.

$$(1 + t^2)x^2 + 2t^2x + t^2 - 1 =$$

$$(1 + t^2)x^2 + (1 + t^2)x - (1 + t^2)x + 2t^2x + t^2 - 1 =$$

Stereographic projection



$(0, t) \mapsto (x, y)$, where (x, y) is the intersection point of $x^2 + y^2 = 1$ and $y = t(x + 1)$.

Substitute:

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$$(1 + t^2)x^2 + 2t^2x + t^2 - 1 = 0$$

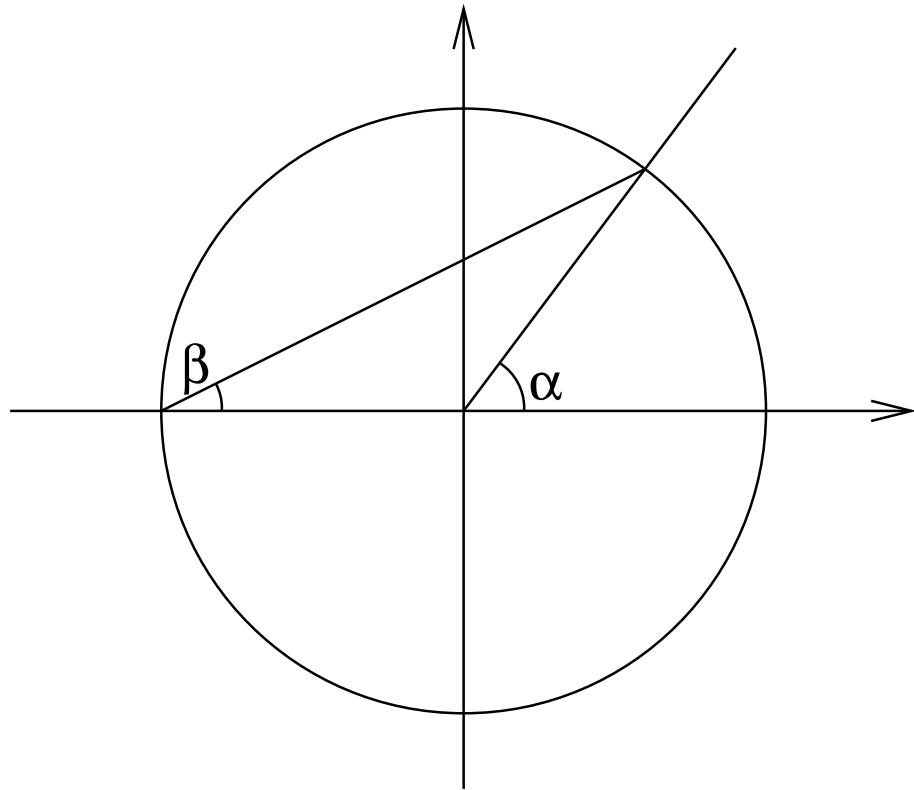
Divide by $x + 1$.

$$(1 + t^2)x^2 + 2t^2x + t^2 - 1 =$$

$$(1 + t^2)x^2 + (1 + t^2)x - (1 + t^2)x + 2t^2x + t^2 - 1 =$$

$$(1 + t^2)x(x + 1) + (t^2 - 1)(x + 1) = (x + 1)((1 + t^2)x - (t^2 - 1)).$$

Stereographic projection



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Divide by $x + 1$.

$$(1 + t^2)x^2 + 2t^2x + t^2 - 1 =$$

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$$(1 + t^2)x(x + 1) + (t^2 - 1)(x + 1) = (x + 1)((1 + t^2)x - (t^2 - 1)).$$

$$x = \frac{1-t^2}{1+t^2}, y = t(x + 1) =$$

Problem 1.1

Find all rational solutions of the equation

$$x^2 + y^2 + 3 = 0.$$

Problem 1.2

Find all rational solutions of the equation

$$x^2 + y^2 - 3 = 0.$$

Problem 1.3

Find all rational solutions of the equation

$$x^2 - xy + y^2 - 4x + 2y + 4 = 0.$$

Problem 2.1

Find all integer solutions of the following equation

$$x^2 + y^2 - 5z^2 = 0.$$

Problem 2.2

Find all integer solutions of the following equation

$$x^2 - 3y^2 + z^2 = 0.$$