Stony Brook University
Mathematics Department
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Introduction to Advanced Mathematics
MAT 150
December 11, 2016

Program of the course

Logic and Set Theory

1. Set and its elements. Axiom of extensionality. The empty set.
2. Describing a set by a list of its elements.
3. Subsets and inclusions. Set-builder notation. Predicates and characteristic functions.
4. Operations on subsets: union, intersection, complement, difference, symmetric difference. Equality of subsets. Proving an equality of subsets by proving of the two opposite inclusions.
5. Logical connectives: negation, conjunction, disjunction, implication, and equivalence. Logical symbols.
6. Relations between logical connectives and operations on subsets.
7. Propositional forms. Truth tables. Constructing and analysing propositional forms. Logical identities, in particular: tautology, contradiction, de Morgan's laws, the law of excluded middle and the law of consistency.
8. A variety of colloquial expressions for logical connectives. For instance: and, but, though and nevertheless correspond to the same connective - conjunction.
9. Quantifiers. Propositional forms with quantifiers. Colloquial expressions associated with quantifiers. Logical identities with quantifiers. In particular, commuting of quantifiers, useful negations of propositions with quantifiers.
10. Maps. Images and pre-images. The main classes of maps: injective, surjective and bijective. Compositions of maps. Identity and Inclusion maps. Inverse map. Equivalence of invertibility and bijectivity.
11. Cartesian products of sets. Maps associated to a Cartesian product and their relation to quantifiers.
12. Conditional and biconditional statements. Colloquial expressions associated with conditionals and biconditionals ("sufficient", "necessary", "sufficient and necessary", " whenever", "if and only if", "iff", etc.)
13. Logical structure of theorems. Proofs, motivations, conjectures, statements and counter-examples.
14. Logical structure of definitions.
15. The contrapositive, the converse, and the inverse of a conditional statement.
16. Types of proofs: direct proof, proof by contraposition, proof by contradiction, proof by exhaustion.
17. Typical logical mistakes of affirming the consequent and denying the antecedent.
18. Proofs using principle of mathematical induction in different forms (induction, strong induction, well-ordering principle).

19. Relations on a set. Properties of binary relations: reflexivity, irreflexivity, symmetry, antisymmetry, transitivity.
20. Partial order and linear order, their strict and non-strict versions.
21. Equivalence relations, equivalence classes, quotient set and the theorem about one-to-one correspondence between equivalence relations on a set and partitions of the set.
22. Congruence arithmetic.
23. Equipotency of sets. Its properties (reflexivity, symmetry and transitivity). The notion of cardinal numbers. See the texts on BlackBoard: “Chapter Set theory from R&T”, “SetStories” and wikipedia:

    `https://en.wikipedia.org/wiki/Cardinal_number`
24. Arithmetic operations with cardinal numbers. See

    `https://en.wikipedia.org/wiki/Cardinal_number`
25. Cardinality of $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$.
26. Properties of finite sets. Pigeon hole principle.
27. Denumerable sets and their properties.
28. Cantor theorem about cardinalities of a set and its power set.
29. The notion of inequality of cardinal numbers. Transitivity. Cantor-Schröder-Bernstein Theorem; see wikipedia,

    `https://en.wikipedia.org/wiki/Schroeder-Bernstein_theorem`

    and trichotomy property.
30. Uncountability of $\mathbb{R}$.

Other topics

1. Axioms of distance. Metric spaces. Balls and spheres in a metric space.
2. Neighborhoods of a point in a metric space.
3. Continuity at a point of a map between metric spaces: definition in terms of neighborhoods and epsilon-delta definition and their equivalence.
4. Axioms of topological structure. Open sets. Neighborhoods of points.
5. Metric topology.
6. Inner, exterior and boundary points of a set in a topological space.
7. Pythagorean triples. Integer solutions of the equation $x^2 + y^2 = z^2$.
8. Rational points on the circle $x^2 + y^2 = z^2$.
9. Diophantine equations of degree 2.
10. Squares of natural numbers modulo 2, 4 and 8.
11. Constructions of plane geometric figures by the loci method.
12. Isometries of the Euclidean plane. Their presentations as compositions of reflections in lines.