Stony Brook University
Mathematics Department
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MAT 150
**Introduction to Advanced Mathematics**
Fall 2016

# Practice Midterm 1

This Practice Midterm is given to show what kind of problems you may expect on the exam. It is proposed to you as a homework, due by Thursday 10/13. Actual Midterm 1 will be shorter and easier.

**1.** True or false? Explain! ($a, x$ are real numbers).

**a)** $\forall a \, \exists x \, (x^2 + ax - 1 = 0)$ **b)** $\exists x \, \forall a \, (x^2 + ax - 1 = 0)$ **c)** $\forall x \, \exists a \, (x^2 + ax - 1 = 0)$.

**2.** Ken Olsen, CEO of *Digital Equipment*, claimed in 1977:

"There is no reason for any individual to have a computer in his home"

Give a symbolic writing of this phrase. You have to describe the universes, introduce appropriate notations and use quantifiers.

**3.** Use $\varepsilon$-$\delta$ definition to prove that $\lim_{x \to 1}(5 - 2x) = 3$.

**4.** A real number $L$ is called a *limit* of a sequence $\{a_n\}_{n=1}^{\infty}$ if for any positive real number $\varepsilon$ there exists a positive integer $N$ such that for all integers $n$ greater than $N$ the inequality $|a_n - L| < \varepsilon$ holds true. Notation: $L = \lim_{n \to \infty} a_n$.

**a)** Rewrite this definition in a symbolic form.

**b)** Use this definition to prove that $\lim_{n \to \infty} \frac{1}{n} = 0$.

**c)** Explain what it means that a number $L$ is **not** the limit of the sequence $\{a_n\}_{n=1}^{\infty}$.

**4.** Give definition of a composite number, both in words and in symbols. Remember that 1 is neither prime nor composite.

**5.** Give definition of an even function. Give definition of an odd function. What does it mean that a function is neither even nor odd? (Give a symbolic description.) For each of the following functions, use the definitions to determine whether it is even or odd or neither: $f(x) = x \sin x$, $g(x) = x^3 + 1$, $h(x) = e^x - e^{-x}$.

**6.** Prove or give a counterexample:

**a)** There do not exist three consecutive integers $a, b, c$ such that $a^2 + b^2 = c^2$.

**b)** There do not exist three consecutive even integers $a, b, c$ such that $a^2 + b^2 = c^2$.

**c)** There do not exist three consecutive odd integers $a, b, c$ such that $a^2 + b^2 = c^2$.

**7.** Prove that for all real numbers $x$, $\dfrac{3|x-2|}{x} \leq 4$ whenever $x \geq 1$.

**8.** Formulate and prove the triangle inequality.

**9.** For Fibonacci sequence $a_n$, prove the following:

**a)** $a_1 + a_3 + \cdots + a_{2n-1} = a_{2n}$

**b)** $\sum_{i=1}^{n} i a_i = n a_{n+2} - a_{n+3} + 2$.

**10.** Prove statements which are true and give counterexamples for those which are false.

**a)** $A = B \cup C$ is sufficient for $A \setminus B \subset C$.
**b)** $A = B \cup C$ is necessary for $A \setminus B \subset C$.