

1. Let X and Y be sets. Prove that an injective map $X \rightarrow Y$ exists if and only if a surjective map $Y \rightarrow X$ exists.

2. Let X and Y be sets, $X_1 = X \amalg \{a\}$ and $Y_1 = Y \amalg \{b\}$,
(i.e., X_1 and Y_1 , respectively, are disjoint sums of X and Y , respectively, and sets consisting of a single element a and b).

(1) Prove that if there exists a surjective map $X_1 \rightarrow Y_1$, then there exists a surjective map $X \rightarrow Y$.

(2) Prove that if there exists an injective map $X_1 \rightarrow Y_1$, then there exists an injective map $X \rightarrow Y$.

3. Pigeon Hole Principle says that if p pigeons sit in q pigeon holes and $p > q$ then there exists a hole in which more than one pigeons sit.

(1) Translate this principle into the language of sets and maps.

(2) Prove the statement about sets and maps that you have just found.

(3) Is this statement applicable to infinite sets?