MAT 150, Introduction to Advanced Mathematics Name ________________________________

Homework 2, due by 9/22

Score______________

1. Formulate a necessary condition for a local extremum of a real valued function in one variable.

Is this condition sufficient?

2. Is it true that for real numbers x and y and natural number n the inequality $x < y$ is necessary and sufficient for $x^n < y^n$?

Formulate correct statements about relations between these two inequalities.

3. Which of the following are true in the universe of all real numbers?

$(\forall x)(\exists y)(x + y = 0)$.
$(\exists x)(\forall y)(x + y = 0)$.
$(\exists x)(\exists y)(x^2 + y^2 = -1)$.
$(\forall x)[x > 0 \Rightarrow (\exists y)(y < 0 \wedge xy > 0)]$.
$(\forall y)(\exists x)(\forall z)(xy = xz)$.
$(\exists x)(\forall y)(x \leq y)$.
$(\forall y)(\exists x)(x \leq y)$.
$(\exists! y)(y < 0 \wedge y + 3 > 0)$.
$(\exists! x)(\forall y)(x = y^2)$.
$(\forall y)(\exists! x)(x = y^2)$.
$(\exists! x)(\exists! y)(\forall w)(w^2 > x - y)$.

4. Write the symbolic form of the statement of the Mean Value Theorem.

5. Which of the following are denials of $\exists! x P(x)$?

$(\forall x)P(x) \vee (\forall x)\sim P(x)$.
$(\forall x) \sim P(x) \vee (\exists y)(\exists z)(y \neq z \wedge P(y) \wedge P(z))$.
$(\forall x)[P(x) \Rightarrow (\exists y)(P(y) \wedge x \neq y)]$.
$\sim (\forall x)(\forall y)[(P(x) \wedge P(y)) \Rightarrow x = y]$.