MAT 315, Advanced Linear Algebra Name ____________________

**Homework 6**

due by April 2 Score__________

**Problem 1.** Let $V$ be a vector space over a field $\mathbb{F}$ and let $T : V \to V$ be a linear operator such that $T^2 = I$.
(a) Prove that if the characteristic of $\mathbb{F}$ is not 2, then $V$ is direct sum of two subspaces, $V_+$ and $V_-$ invariant under $T$ such that $T|_{V_+} = I$ and $T|_{V_-} = -I$.
(b) Under assumptions of (a), is $T$ diagonolizable?
If so, what is its diagonalization?
(c) Prove that if $\mathbb{F} = \mathbb{Z}/_2$ and $V = \mathbb{F}^2$, then there exists $T : V \to V$ which is not diagonalizable.

**Problem 2.** Let $T$ be a linear operator over $\mathbb{F}$ such that $T^3 = \mathrm{id}$. Prove that
(a) if $\mathbb{F} = \mathbb{C}$ then $T$ is diagonalizable
(b) if $\mathbb{F} = \mathbb{R}$ and $T$ is diagonalizable, then $T = \mathrm{id}$.

**Problem 3.** Let $V$, $W$ be vector spaces over a field $\mathbb{F}$.
(a) Write down a definition for $\phi \otimes \psi \in \mathcal{L}(V, W; \mathbb{F})$ if $\phi \in V^{\checkmark}$ and $\psi \in W^{\checkmark}$.
(b) Write down a definition of $a \wedge b \in \Lambda^{p+q}(V)$, where $a \in \Lambda^p(V)$ and $b \in \Lambda^q(V)$.

**Problem 4.** Let $e_1, e_2, \ldots, e_{2n}$ be a basis of a vector space $V$ and $\omega = e^1 \wedge e^2 + e^3 \wedge e^4 + \cdots + e^{2n-1} \wedge e^{2n}$.
Prove that $\underbrace{\omega \wedge \omega \wedge \cdots \wedge \omega}_{n \text{ times}} = n! e^1 \wedge e^2 \wedge \cdots \wedge^{2n}$.