MAT 315, Advanced Linear Algebra Name _____

Homework 4

due by March 5 Score_____

1. The vector space $\mathcal{P}_m(\mathbb{R})$ of real polynomials of degree $\leq m$ in variable x has a basis $1, x, \ldots, x^m$.
   (a) Find explicit formulas for functionals which form the dual basis.
   (b) Do the same for a basis $1, x-1, (x-1)^2, \ldots, (x-1)^m$.

2. Let V, W be finite-dimensional vector spaces over a field $\mathbb{F}$. Prove that the map
$$\mathcal{L}(V,W) \to \mathcal{L}(W^\vee, V^\vee) : T \to T^\vee$$
is an isomorphism of a vector space $\mathcal{L}(V,W)$ to a vector space $\mathcal{L}(W^\vee, V^\vee)$.

3. Let V, W be finite-dimensional vector spaces over a field $\mathbb{F}$ and $T : V \to W$ be a linear map.
   (a) Find formulas expressing numbers $\operatorname{rk} T^\vee$, $\dim \operatorname{null} T^\vee$ in terms of $\operatorname{rk} T$ and $\dim \operatorname{null} T$.
   (b) Find isomorphisms which justify the formulas which you found in (a).

4. What can you say about $T^\vee$ if T is
   (a) injective,
   (b) surjective?