MAT 315, Advanced Linear Algebra Name ______________________

Homework 3

due by February 27 Score __________

1. For any vector space V over a field $\mathbb{F}$, construct an isomorphism $S_V : V \to \mathcal{L}(\mathbb{F}, V)$ (don't forget to prove that this is an isomorphism indeed).

2. Let U, V, W be finite-dimensional vector spaces.
(a) If any linear map $T : U \to W$ can be presented as a composition of linear maps $U \to V \to W$, then what can be the dimensions of the spaces?
(b) Which linear maps $U \to W$ can be presented as compositions $U \to V \to W$?

3. Let V, U and W be finite-dimensional vector spaces over a field $\mathbb{F}$ and $p : V \to U$ and $q : V \to W$ be linear surjective maps such that $V = \operatorname{null} p \oplus \operatorname{null} q$.
(a) Prove that V is isomorphic to $U \oplus W$.
(b) Construct an isomorphism explicitly.
(c) Are the spaces V and $U \oplus W$ still isomorphic if they are not assumed to be finite-dimensional?