MAT 310/315, Linear Algebra Name _____

Homework 1

due 2/6 Score_____

1. Suppose a, b, c, d are real numbers. Find real numbers x, y such that

$$(x + yi)(a + bi) = c + di$$

2. Explain why there does not exist $\lambda \in \mathbb{C}$ such that

$$\lambda(2 - 3i, -6 + 7i, 5 + 4i) = (12 - 5i, -32 - 9i, 7 + 22i)$$

3. For each of the following subsets of $\mathbb{F}^3$, determine whether it is a subspace of F^3:
(a) $\{(x_1, x_2, x_3) \in \mathbb{F}^3 \mid x_1 + 2x_2 = 3x_3\}$;
(b) $\{(x_1, x_2, x_3) \in \mathbb{F}^3 \mid x_1 + 2x_2 + 3x_3 = 5\}$;
(c) $\{(x_1, x_2, x_3) \in \mathbb{F}^3 \mid x_1 x_2 = 0\}$

4. Is it true that if a non-empty subset $U \subset \mathbb{R}^2$ is closed under addition (i.e., $U + U \subset U$) and under multiplication by -2 (i.e., $(-2)U \subset U$), then U is a vector subspace of $\mathbb{R}^2$?

If your answer is YES, then prove it, if your answer is NO, then give a counter-example.

5. Prove that the union of two subspaces of V is a subspace of V if and only if one of the subspaces is contained in the other.

6. Prove or give a counterexample: if U, W_1, W_2 are subspaces of V such that

$$V = U \oplus W_1 \text{ and } V = U \oplus W_2$$

then $W_1 = W_2$.