Remarks on Lecture 4

Quotient spaces. In the beginning we studied quotient spaces. Two pages are available inthe format of handouts, file Lect-04H.pdf. This material can be found on pages 94-96 of the textbook.

After that we considered the quotient map $\pi : V \to V/U$ as in Definition 3.88, established a few facts on π:

π is linear map;
$\operatorname{null} \pi = U$;
π is surjective, so $\operatorname{rk} \pi = \dim V/U$;
$\operatorname{rk} \pi + \dim \operatorname{null} \pi = \dim V$;
$\dim V = \dim U + \dim V/U$, so $\dim V/U = \dim V - \dim U$.

Then look Definition 3.90 of injective quotient $\tilde{T} : V/\operatorname{null} T \to W$ for a linear map $T : V \to W$. Propertes of $\tilde{T}$ in 3.91.

We discuss also a straighrforward generalization of $\tilde{T}$: a linear map $T : V \to W$ and subspaces $A \subset V$, $B \subset W$ such that $T(A) \subset B$ define a linear map $V/A \to W/B$.

Any linear map $T : V \to W$ admits a canonical factorization into the projection $V \to V/\operatorname{null} T$, isomorphism $V/NullT \to \operatorname{range} T$ and inclusion $\operatorname{range} T \to W$. The first map is surjective, the second one is bijective and the last is injective.

Duality. The first part of this section is presented in the textbook. See Definition 3.92, Examples 3.94.

Warning: I will denote the dual space $\mathcal{L}(V, \mathbb{F})$ by $V^\vee$ while in the textbook it is denoted by V'.

Then we discussed Definition 3.96 of the basis of $V^\vee$ dual to a basis of V. It was proven that this is a basis (Theorem 3.98).

A linear map $T^\vee$ dual to a linear map $\mathcal{L}(V, W)$ was defined, see Definition 3.99.

The properties of $T^\vee$ were established, Theorem 3.101.

At the last minutes of lecture, the notion of functor was introduced, see https://en.wikipedia.org/wiki/Functor