MAT 200 Logic, Language and Proof Name ______________________

Homework 2

Score__________

1. (8pt)
Let $\mathbb{N}$ be the set of positive integers. Determine the truth value of the following statements. Justify your answers.

(1) $\forall p \in \mathbb{N}\ \forall q \in \mathbb{N}\ p \leq q$

(2) $\exists p\ \exists q \in \mathbb{N}\ p \leq q$

(3) $\forall p \in \mathbb{N}\ \exists\ q \in \mathbb{N}\ p \leq q$

(4) $\exists p \in \mathbb{N}\ \forall q \in \mathbb{N}\ p \leq q$

(5) $\exists p \in \mathbb{N}\ \forall q \in \mathbb{N}\ p < q$

(6) $\forall q \in \mathbb{N}\ \exists p \in \mathbb{N}\ p \leq q$

(7) $\forall q \in \mathbb{N}\ \exists p \in \mathbb{N}\ p < q$

(8) $\exists q \in \mathbb{N}\ \forall p \in \mathbb{N}\ p \leq q$

2. (6pt)
Construct all possible quantified sentences using the predicate $y < 1 - |x|$ (the universe is $\mathbb{R}$). For each of the sentences, give the truth value.

3. (4pt)
Show on the coordinate line all the values of variable x for which the implication $(x \in [0, 3]) \implies (x \in [2, 7])$ holds true.

4. (3pt)
State in affirmative terms (without $\neg$) and give the truth value.

(1) $\neg\ (\exists x\ x^2 < 0)$

(2) $\neg\ (\forall x\ \exists y\ x^2 + y^2 \leq 1)$

(3) $\neg\ (\exists y\ \forall x\ x^2 + y^2 > 3)$

5. (4pt)
Give definition of even function defined on $\mathbb{R}$.

Give definition of odd function defined on $\mathbb{R}$.

Using these definitions explain which functions are neither even, nor odd.

On the basis of your definitions explain which of the following functions is even, odd or neither:
$f(x) = 3x^2 + 1,\ \ g(x) = x^3 + 3x,\ \ h(x) = x^2 - x.$