MAT 200 Logic, Language and Proof   Name _____

Homework 1

Score_____

1. (5pt) Write down the truth table for the propositional forms $P \wedge (Q \vee R)$ and $(P \wedge Q) \vee (P \wedge R)$. What does the result suggest?

2. (6pt)

(a) Is a proposition P true if the proposition $P \wedge (2 \cdot 2 \leq 5)$ is true? Justify your answer.

(b) Is a proposition P true if the proposition $P \vee (3 \cdot 3 > 7)$ is true? Justify your answer.

3. (10pt) Are the following propositions true or false. Justify your answers.

(1) $\forall x \ ((x^2 - 3x + 18 \geq 0) \implies (x \leq 5))$

(2) $\forall x \ ((x^2 - 2x + 8 \geq 0) \implies (x \leq 5))$

(3) $\forall x \ ((x \geq 5) \implies (10 + 3x - x^2 \leq 0))$

(4) $\forall x \ (\sqrt{x^2} = x)$

(5) $\exists x \ (\sqrt{x^2} = -x)$

(6) $\exists x \ (\sqrt{x^2} = x)$

(7) $\forall x \ (\sqrt{x^2} = -x)$

(8) $\forall x \ (\sqrt{x^2} = |x|)$

(9) $|x| = -x$ iff $\sqrt{x^2} = -x$ (x is a real number).

(10) 14 is a composite number if $2 \cdot 7 \neq 14$.

4. (4pt) On a paper, 25 statements are written:

There is exactly one false statement on this paper.
There are exactly two false statements on this paper.
There are exactly three false statements on this paper.
. . . .
There are exactly 25 false statements on this paper.

Are there any true statements among those 25 statements? Justify your answer!