# MAT 312 - AMS 351 FINAL EXAM

SUMMER II, 29 August 2014

NAME: 

ID:

ANSWER ALL QUESTIONS.

SHOW YOUR CALCULATIONS

DO NOT TEAR-OFF ANY PAGE

NO CALCULATORS NO CELLS NO NOTES ETC.

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Question 1.

a. Find the largest prime divisor of $42! + 43! + 44!$. (7 pts)

b. Show that 495 divides $21^{240} - 36^{120}$. (8 pts)

c. Find the greatest common divisor of 1365 and 4264 using Euclidean Algorithm. (5 pts)
Question 2. Consider the permutations

\[ \pi = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 4 & 1 & 5 & 2 \end{pmatrix} \]

\[ \sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 3 & 4 & 2 & 1 \end{pmatrix} \]

a) Calculate the order, sign of the permutations \( \pi \), \( \sigma \) and \( \pi \sigma \) (show the formula that you use in each case). (10 pts)

b) Find an element \( \tau \) in \( S(5) \) satisfying

\[ \tau \pi \tau^{-1} = \sigma. \]

[Hint: You may assume that \( \tau \) is given by \( \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ a & b & c & d & e \end{pmatrix} \) and solve \( a, b, c, d, e. \) (10 pts)
Question 3. Let $p$ and $q$ be two distinct primes. Consider the group $(\mathbb{Z}_p, +)$ and the map

$$\varphi: \mathbb{Z}_p \rightarrow \mathbb{Z}_p$$

$$x \mapsto qx \mod p.$$  

Show that $\varphi$ is an injective group homomorphism. \hspace{1cm} (15 pts)
Question 4.

a. What is the order of the group \((\mathbb{Z}_n \times \mathbb{Z}_m, +)\)? [The group operation is given by \((a, b) + (c, d) = (a + c, b + d)\) for all \((a, b), (c, d) \in \mathbb{Z}_n \times \mathbb{Z}_m\).] \(6 \text{ pts}\)

b. Define the subset \(L\) of \(\mathbb{Z}_n \times \mathbb{Z}_m\) by
\[
L = \{(x, y) \in \mathbb{Z}_n \times \mathbb{Z}_m \mid x - y = 0\}.
\]
Show that \(L\) is a subgroup of \(\mathbb{Z}_n \times \mathbb{Z}_m\). \(10 \text{ pts}\)

c. Assume that \(n \leq m\). Determine the number of distinct cosets of \(L\) in \(\mathbb{Z}_n \times \mathbb{Z}_m\). \(9 \text{ pts}\)
Question 5.

a. Let $f$ be a linear code function generated by the matrix

$$
\begin{pmatrix}
1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 \\
0 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \\
0 & 0 & 1 & 1 & 1 & 0 & 1 & 0 
\end{pmatrix}.
$$

Using the corresponding parity check matrix, determine whether the following are codewords or not.

11101000, 01110111, 10001001.

(10 pts)

b. Find an example of a linear code $f : \mathbb{B}^4 \rightarrow \mathbb{B}^8$ with minimum distance 4 or prove that such code does not exist.

(10 pts)