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Midterm 2 Review

Here are some review problems for the second midterm. Please send me an email if you find any errors.

1. A coffee filter in the shape of an inverted cone (8 inches in diameter and 4 inches tall) is dripping coffee. There is a cylindrical coffee pot (10 inches in diameter and 6 inches tall) below the cone. How fast is the height of the coffee in the pot increasing when the level of coffee in the filter is 2 inches and decreasing at a rate of .2 inches per second?
2. A lighthouse is located on a small island 5 km away from the nearest point P on a straight shoreline. Suppose its light makes 8 revolutions per minute. How fast is the beam of light moving along the shoreline when it is 2 km away from P?
3. Suppose $f'(x) = (\ln x) \cdot e^{-x^2}$.
   (a) On what interval is f increasing? On what interval is f decreasing?
   (b) Does f have a maximum value? Minimum value?
4. Give an example of a continuous function f such that
   (a) f' is always positive and f'' is always negative.
   (b) f has a minimum at some point c but $f''(c)$ is not positive.
   (c) f has a horizontal tangent but no local maximum or minimum.
5. Let $g(\theta) = 2\sin\theta + \sin^2\theta$ on the interval $0 \leq \theta \leq 2\pi$.
   (a) Find the critical values and determine whether or not they are local maximum/minimum.
   (b) Find the intervals of concavity and the inflection points.
   (c) What is the absolute maximum/minimum?
6. An inverted square pyramid with height h is inscribed in a larger square pyramid with height H so that the vertex is at the center of the base of the larger pyramid. Show that the inner pyramid has maximum volume when $h = \frac{1}{3}H$.
7. At which points on the curve $y = 10 + 40x^3 - 3x^5$ does the tangent line have the largest slope?