Problem 1.

Suppose that $X$ and $Y$ are disjoint sets. One can prove that the function
\[
\bigcup_{j=0}^{k} \mathcal{P}_j(X) \times \mathcal{P}_{k-j}(Y) \rightarrow \mathcal{P}_k(X \cup Y)
\]
defined by \((A, B) \mapsto A \cup B\) is a bijection. Taking this for granted, deduce that
\[
\binom{m+n}{k} = \sum_{j=0}^{k} \binom{m}{j} \binom{n}{k-j}.
\]

Problem 2.

Prove that for $n \in \mathbb{N}$,
\[
\sum_{j=0}^{n} (-1)^j \binom{n}{j} = 0.
\]

Problem 3.

What is the sum of the coefficients of the polynomial \((1 + x)^{53}\)?