Due in Class: October 7, 2015.

Reading: Read Chap.10

Turn in the following exercises.

**Problem 1.**
Let $f : X \rightarrow Y$ be a function. Prove that there exists a function $g : Y \rightarrow X$ such that $f \circ g = I_Y$ if and only if $f$ is surjective. Here $I_Y$ denotes the identity function on $Y$.

**Problem 2.**
Let $f : X \rightarrow Y$ be a function, let $A_1, A_2$ be subsets of $X$ and $B_1, B_2$ be subsets of $Y$.

1. Prove that $A_1 \subseteq A_2 \Rightarrow f(A_1) \subseteq f(A_2)$, but that the converse is false in general. Is the converse true if $f$ is injective? Explain.
2. Prove that $B_1 \subseteq B_2 \Rightarrow f^{-1}(B_1) \subseteq f^{-1}(B_2)$, but that the converse is false in general. Is the converse true if $f$ is surjective? Explain.
3. Prove that $f(A_1 \cap A_2) \subseteq f(A_1) \cap f(A_2)$. Give an example for which equality does not hold.
4. Prove that $f(A_1 \cup A_2) = f(A_1) \cup f(A_2)$.

**Problem 3.**
Suppose that there are 153 students enrolled in at least one of the three first year core Mathematics courses (Logic, Algebra and Calculus). If 100 of these students like Logic, 100 like Algebra, 100 like Calculus, 56 like Logic and Algebra, 60 like Logic and Calculus, 57 like Algebra and Calculus, and 25 like all three courses, how many of the students like none of the courses?

**Problem 4.**
At a Mathematics Conference of 100 participants, 75 speak English, 60 speak Spanish and 45 speak Italian, and everyone present speaks at least one of these languages.

1. What is the maximum number of participants who can speak only one language?
2. What is the maximum number of participants who speak only English?
(3) Prove that the greater the number of participants who speak all three languages, the greater the number of participants who speak only one language.