Due in Class: September 30, 2015.

Reading: Read Chap.9

Problem 1.
Let $X$ be any set. Given $A \in \mathcal{P}(X)$, we define the characteristic function of $A$, $\chi_A : X \to \{0, 1\}$, by

$$\chi_A(x) = \begin{cases} 0 & \text{if } x \notin A \\ 1 & \text{if } x \in A \end{cases}$$

for $x \in X$.

Suppose now that $A$ and $B$ are two subsets of $X$.

1. Prove that $\chi_{A \cap B}(x) = \chi_A(x)\chi_B(x)$ for all $x \in X$.
2. Prove that $\chi_{A \cup B}(x) = \chi_A(x) + \chi_B(x) - \chi_A(x)\chi_B(x)$ for all $x \in X$.

Problem 2.
Define functions $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ by

$$f(x) = \begin{cases} x + 2 & \text{if } x < -1 \\ -x & \text{if } -1 \leq x \leq 1 \\ x - 2 & \text{if } x > 1 \end{cases}$$

$$g(x) = \begin{cases} x - 2 & \text{if } x < -1 \\ -x & \text{if } -1 \leq x \leq 1 \\ x + 2 & \text{if } x > 1 \end{cases}$$

1. Find the functions $f \circ g$ and $g \circ f$.
2. Is $g$ the inverse of the function $f$? Explain.
3. Is $f$ injective or surjective? Explain.
4. Is $g$ injective or surjective? Explain.

Problem 3.
Let $X$ be a set with three elements, say $X = \{a, b, c\}$. Find all bijective functions from $X$ to $X$.

Problem 4.
Let $X, Y, Z$ be sets and suppose that $f : X \to Y$ and $g : Y \to Z$ are surjective functions. Prove that the composition $g \circ f : X \to Z$ is surjective.