Due in Class: September 16, 2015.

Reading: Read p.61–99.

Turn in the following exercises.

Problem 1.
Prove by induction on \( n \) that \( n! > 2^n \) for all integers \( n \geq 4 \).

Problem 2.
Prove by induction on \( n \) that
\[
\sum_{j=1}^{n} \frac{1}{j(j+1)} = \frac{n}{n+1}
\]
for all positive integers \( n \).

Problem 3.
For a positive integer \( n \), the number \( a_n \) is defined inductively by
\[
a_1 = 1
\]
and
\[
a_{k+1} = \frac{6a_k + 5}{a_k + 2}
\]
for \( k \) a positive integer. Prove by induction on \( n \) that \( 0 < a_n < 5 \).

Problem 4.
Prove by induction on \( n \) that
\[
\prod_{j=2}^{n} \left(1 - \frac{1}{j^2}\right) = \frac{n + 1}{2n}
\]
for integers \( n \geq 2 \). See Problem 18 p.55 for the inductive definition of the product.