MAT 131 sample problems for test 2

The following problems are not homework.

1.Find the limits.

1)

$$\lim_{x\to+\infty}\frac{3x^2+5x-2}{4x^2-9}=\frac{3}{4}$$

2)

$$\lim_{x\to+\infty}(\sqrt{x^2+1}-x)=\lim_{x\to+\infty}\frac{(\sqrt{x^2+1}-x)\cdot(\sqrt{x^2+1}+x)}{\sqrt{x^2+1}+x}=\lim_{x\to+\infty}\frac{1}{\sqrt{x^2+1}+x}=0$$

3)

$$\lim_{x\to0^-}2^{\frac{1}{x}}=0$$

4)

$$\lim_{x\to1}(\frac{1}{x-1}+\frac{2}{x^2-4x+3})=\frac{1}{x-3}=\frac{-1}{2}$$

5)

$$\lim_{x\to\pi^-}\ln(\sin x)=-\infty$$

6)

$$\lim_{x\to-\infty}(\sqrt{x^2-x}-\sqrt{x^2+4x+1})=\lim_{x\to-\infty}\frac{(\sqrt{x^2-x}-\sqrt{x^2+4x+1})\cdot(\sqrt{x^2-x}+\sqrt{x^2+4x+1})}{\sqrt{x^2-x}+\sqrt{x^2+4x+1}} \tag{1}$$

$$=\lim_{x\to-\infty}\frac{(x^2-x)-(x^2+4x+1)}{\sqrt{x^2-x}+\sqrt{x^2+4x+1}} \tag{2}$$

$$=\lim_{x\to-\infty}\frac{-1-5x}{\sqrt{x^2-x}+\sqrt{x^2+4x+1}} \tag{3}$$

$$=\lim_{x\to-\infty}\frac{\frac{(-1-5x)}{-x}}{\frac{\sqrt{x^2-x}+\sqrt{x^2+4x+1}}{\sqrt{x^2}}} \tag{4}$$

$$=\frac{5}{2} \tag{5}$$

7)

$$\lim_{x\to-\infty}(e^x\cdot\sin x)=0$$

8)

$$\lim_{x\to 0}\frac{\sin(x^2)}{x^2}=1$$

9)

$$\lim_{x\to 0}\frac{(\sin x)^2}{x^2}=1$$

10)

$$\lim_{x\to +\infty}\frac{x^3}{2^x}=0$$

11)

$$\lim_{x\to +\infty}\arctan x=\frac{\pi}{2}$$

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$$\lim_{x\to 0}\frac{3^x-1}{2^x-1}=\lim_{x\to 0}\frac{\frac{3^x-1}{x-0}}{\frac{2^x-1}{x-0}}=\frac{\ln 3}{\ln 2}$$

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$$\lim_{x\to \pi/3}\frac{\cos x-\frac{1}{2}}{x-\frac{\pi}{3}}=\frac{d(\cos x)}{dx}\Big|_{x=\pi/3}=-\sin\frac{\pi}{3}=-\frac{\sqrt{3}}{2}$$

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$$\lim_{x\to 0}x\cdot\sin\frac{1}{x^2}=0$$

2.Find the derivatives.

1)

$$f(x)=\frac{(x^2+4x+3)\cdot x^{\frac{4}{3}}}{(x^3-1)^4}$$

$$f^{'}(x)=\frac{(x^2+4x+3)\cdot x^{\frac{4}{3}}}{(x^3-1)^4}\cdot(\frac{2x+4}{x^2+4x+3}+\frac{4}{3x}-\frac{12x^2}{x^3-1})$$

2)

$$f(x)=e^{\cos x}$$

$$f^{'}=-e^{\cos x}\sin x$$

3)

$$f(x)=\arctan(\ln(x^2+1))$$

$$f^{'}=\frac{1}{1+(\ln(x^2+1))^2}\cdot\frac{2x}{x^2+1}$$

4)

$$f(x) = \sin(x + \cos x)$$

$$f^{'}(x) = \cos(x + \cos x) \cdot (1 - \sin x)$$

5)

$$f(x) = e^{x \cdot (\tan x)^2}$$

$$f^{'} = e^{x \cdot (\tan x)^2} \cdot ((\tan x)^2 + 2x \tan x(1 + (\tan x)^2))$$

6)

$$f(x) = \sqrt[4]{x + \sqrt[3]{x}}$$

$$f^{'}(x) = \frac{1}{4}(x + \sqrt[3]{x})^{-3/4} \cdot (1 + \frac{1}{3}x^{-\frac{2}{3}})$$

3. Find $y^{'}$ (The results should be in terms of x and y .)

1)

$$x^4 y + xy^2 = x + 3y$$

2)

$$x^2 \cos y + \sin(2y) = xy$$

3)

$$xe^y = y - 1$$

4)

$$\sin(xy) = x^2 - y$$

4. Sketch the graph for function:

1)

$$f(x) = x^3 - 2x^2 + x + 1$$

2)If we know that $f^{'}(x) = \frac{x}{1+x^2}$ and $f(0) = 0$.

3)

$$f(x) = e^{\sin x}, x \in (0, \pi)$$

5 Find the tangent lines at given points.

1)$x = \ln t$,$y = t^2 + 1$, $(0, 2)$

2)$x = t^3 - 2t^2 + t + 1, y = t^2 + t, (1, 0)$

6.Use a linear approximation to estimate the given number.

1)$(8.006)^{2/3}$

HINT:$f(x) = x^{\frac{2}{3}}, a = 8$

2)1/1001

HINT:$f(x) = \frac{1}{x}, a = 1000$