MAT 200 Midterm 3
Spring 2021

April 30, 2021

Question	Points possible	Score
1	20	
2	10	
3	10	
4	10	
Total	50	

Instructions

1. There are 4 problems on this exam.
2. You have 55 minutes to take the exam.
3. No notes, books, calculators or other resources are allowed, with the exception of a 5×8 index card.
4. Keep bags, coats and other personal belongings at some distance from your seat.
5. You may ask me for clarification on any question, although I may not be able to answer.

Exam

1. (7 questions) Short answer.

(a) (2 points) Mark all of the sets that are **countable**.

$\boxed{\mathbb{Z}\text{, the set of integers}}$ $\boxed{\mathbb{Q}\text{, the set of rationals}}$ $\boxed{\mathbb{R}\text{, the set of real numbers}}$ $\boxed{\mathbb{Z}\times\mathbb{Z}}$

(b) (4 points) Express $0.1717\overline{17}$ as a fraction.

(c) (2 points) Let $a=-10$, $b=7$. Write a in the form $a=qb+r$, where $q\in\mathbb{Z}$ and $r\in\{0,1,\ldots,6\}$, as in the division theorem.

(d) (4 points) Find $\gcd(54,132)$. Hint: use the Euclidean algorithm.

(e) (2 points) January 1, 2021 is a Friday. What day of the week is January 1, 2022? (A year has 365 days.)

(f) (3 points) Find an integer solution to the congruence $7x \equiv 8 \mod 9$ if a solution exists. If no solution exists, explain why.

(g) (3 points) Which congruence classes $[a]_{12}$ modulo 12 (where $a \in \{0, 1, \ldots, 11\}$) have a (multiplicative) inverse in $\mathbb{Z}_{12}$?

2. (10 points) Let X be an arbitrary set. Recall that $\mathcal{P}(X)$ is the power set of X.

(a) Prove that there exists an injective function $f\colon X \to \mathcal{P}(X)$.

(b) Prove that no bijective function $g\colon X \to \mathcal{P}(X)$ exists.

(c) Conclude that $|X| < |\mathcal{P}(X)|$ (the cardinality of X is strictly less than the cardinality of the power set of X).

3. (10 points) Find all integer solutions $x \in \mathbb{Z}$ to the linear congruence $60x \equiv 20 \mod 164$.

4. (10 points) Let $a, m \in \mathbb{N}$ be such that $\gcd(a, m) = 1$. Prove that, for all $b_1, b_2 \in \mathbb{Z}$,

$$ab_1 \equiv ab_2 \mod m \quad \text{if and only if} \quad b_1 \equiv b_2 \mod m.$$