MAT 203 Midterm 2
Section 02, Fall 2021

November 17, 2021

Name: ______________________________

Question	Points possible	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
Total	60	

Instructions

1. There are 6 problems on this exam.
2. You have 80 minutes to take the exam.
3. If you need more space, you may continue your solution on the scratch work page and leave a note to direct the grader there.
4. No notes, books, calculators or other resources are allowed.
5. Keep bags, coats and other personal belongings at some distance from your seat.
6. You may ask me for clarification on any question, although I may not be able to answer.

Exam

1. (a) (4 points) Find all critical points of the function $f(x, y) = x^3 - 3x + y^3 - y^2$, where $f(x, y)$ is defined for all $(x, y) \in \mathbb{R}^2$.

(b) (5 point) Use the second derivative test to classify each critical point in (a) as a local minimum, local maximum, or saddle point.

(c) (1 points) Does $f(x, y)$ have a global maximum? Justify your response.

2. Consider the function $f(x, y, z) = x^2 + y^2 + z^2$ together with the constraint $g(x, y, z) = x - 2y + z = 8$.
(a) (4 points) Set up the system of equations given by the method of Lagrange multipliers for the problem of optimizing $f(x, y, z)$ subject to the constraint $g(x, y, z) = 8$

(b) (4 points) Find all solutions (x, y, z) to the system in part (a).

(c) (2 points) Based on your work above, what is the distance from the plane $x - 2y + z = 8$ to the origin?

3. Let R be the planar lamina described by the region in the first quadrant bounded by $y = x$ and $y = x^3$, with constant density $\rho(x, y) = 1$.
(a) (3 points) Find the total mass of the lamina R.

(b) (4 points) Find the center of mass $(\bar{x}, \bar{y})$ of R.

(c) (3 points) Find the radius of gyration $\bar{\bar{x}}$ of R about the y-axis.

4. Consider the region R in $\mathbb{R}^3$ satisfying $0 \leq z \leq \sqrt{x^2+y^2}$ and $x^2+y^2 \leq 3$. Set up an integral for the volume of R in three different ways.

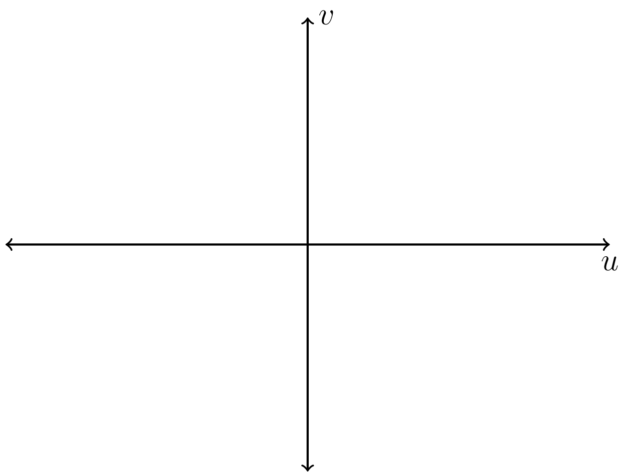
(a) (3 points) In rectangular coordinates.

(b) (3 points) In cylindrical coordinates.

(c) (3 points) In spherical coordinates.

(d) (1 points) Evaluate any one of the three integrals above to compute the volume of R.

5. Let R be the region in the xy-plane whose boundary is the triangle with vertices $(0,0)$, $(2,-1)$, $(1,1)$.
(a) (3 points) Sketch the region S in the uv-plane that gets mapped to R by the change of variables $x = 2u + v$, $y = -u + v$.



(b) (4 points) Write the integral $\iint_R xy\,dA$ as an integral over the region S using the change of variables $x = 2u + v$, $y = -u + v$.

(c) (3 points) Evaluate the integral in part (b).

6. Consider the vector field $\mathbf{F}(x, y, z) = \langle y + z, x + z, x + y \rangle$.
(a) (3 points) Compute curl $\mathbf{F}$.

(b) (1 point) What property does $\mathbf{F}$ have based on your work in part (a)? Explain briefly what this property means.

(c) (4 points) Find a potential function $f(x, y, z)$ for $\mathbf{F}(x, y, z)$.

(d) (2 points) What is the value of the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$, where C is the curve from $(4, 0, 1)$ to $(-4, 0, 1)$ given by $\mathbf{r}(t) = \langle 4\cos t, \sin t, 1 \rangle$, $0 \le t \le \pi$?

[SCRATCH WORK]