Math 331, Spring 2006, Problems - Second set

(Note: `non-Maple` exercises are those for which you *may* not need `Maple` altough you can use it as a word processor.)

16. (`non-Maple`) Proof the Newton's method formula $x_{i+1} = x_i - f(x_i)/f'(x_i)$ following the figures we did in class.

17. Using `Maple` write a program that uses Newton's method to help you find the real roots of the function

$$f(x) = \frac{7}{8}x^5 + \frac{23}{2}x^4 + \frac{459}{8}x^3 + \frac{541}{4}x^2 + 151x + 65$$

Make a judicial choice of initial points by studying the graph of f. Do not use any built-in `Maple` commands to solve equations. (Hint: When you plot the function, select carefully the horizontal and verical ranges.)

18. (a) Explore the situation (with the polynomial of the previous exercise) when the choice of initial value is $x_0 = -3$. Use a for-loop to find the trajectory of this point under Newton's iterations. Explain what happens.

    (b) Explore the situation when $x_0 = -3 + 10^{-9}$ and when $x_0 = -3 - 10^{-9}$. Speculate about how well the method depends on the initial value.

19. (`non-Maple`) Consider the function $f(z) = z^2$. Find out which initial points of the complex plane converge to the (double) root by applying the iterations of Newton's method.

20. For each the following polynomials, describe the basins of attraction of associated Newton functions.

    (a) $p_1(x) = x^2 - 1$.

    (b) $p_2(x) = x^2 + 1$.

    (c) $p_3(x) = x^2 - i$.

    (d) $p_4(x) = x^2 + i$.

    (e) (Extra-credit) $p(x) = x^2 + c$ where c is a real number.

21. For the polynomial $f(x) = \frac{7}{8}x^5 + \frac{23}{2}x^4 + \frac{459}{8}x^3 + \frac{541}{4}x^2 + 151x + 65$

    Make a plot that colors in different colors the points of the complex plane, according to which root of f they approximate by iterating the associated Newton'd method function. Explain the picture obtained. (If you feel like it, you can add the roots of the polynomial to your picture...)

22. For the polynomial $p(z) = z^3 - 2z$ find the fixed points and the points of period two and three. Show how are the points of periods two and three arranged in orbits.*(Hint: Using* fsolve, option complex *may help in the computations.)*

23. Consider the polynomial $P(z) = z^3 + (0.125 + 0.545i)z + 1.125 + 0.545i$.

    (a) Plot (in different colors) the (different) basins of attraction of the associated Newton function.

    (b) Make a procedure that for each point in the complex plane, computes iterates of this point under the Newton function. It should compute at most 50 iterates, but it should stop when an iterate is "close enough" to a root (we will take $abs(f(z)) < 0.1$ as a measure of "close enough"). This procedure should return the number of iterations performed until being "close enough" to a root, or the maximum number of iterations (50).

    (c) Show that the orbit of z=0 does not go to a root under iteration of the associated Newton function. Describe what happens with this orbit.

    (d) Are there more points in the complex plane with an that exhibits similar behavoir (to the orbit of z=0)? If your answer is yes, show an example of an orbit whose behavoir is similar to the behavoir of the orbit of $z = 0$.

24. Consider the quadratic polynomial $P(z) = z^2 + c$ where c is a complex number. For each of the following values of c, first compute the "fate" of the orbit of 0. Second, compute the filled Julia set.

    (a) $c = -0.1 - 0.75i$

    (b) $c = 0.12 - 0.75i$

    (c) $c = 0.451 + 0.21i$

    (d) $c = -0.5 - 0.55i$

    (e) $c = -0.23 + 0.45i$

    Can you see any connection between the fate of the orbit of 0 and the shape of the filled Julia set? (Hint: Check whether different "pieces" of the set seem to be connected to each other or not). You can produce a picture with a grid with a small number of points in Maple and then check a more precise picture in the webpage

    http://math.bu.edu/DYSYS/applets/JuliaIteration.html