Math 331, Spring 2006, Problems

1. Use `Maple` to write $x^5-2x^4-10x^3+20x^2-16x+32$ as a product of *exact* linear factors. By exact, I mean you should leave any non-rational factors expressed as radicals; do not approximate terms like $\sqrt{3}$ as 1.73205, etc.

2. Consider the planar curve γ defined by $x^2y^3+y^2+y-2e^x=0$. Using **only** `Maple`, find the slope of the tangent line to the curve at $(0,1)$. Then plot the curve and the tangent line on the same graph.
   Hint: you might want to use `implicitplot` *from the library* `plots`*. You might find* `implicitdiff` *helpful, too.*

3. Plot the function $f(x)=2\sin x-x^3-5$, for $x\in[-4,4]$. Find all the zeros of the function with an accuracy of 20 decimal digits. *Hint: See* `Digits`*,* `fsolve`*, and/or* `evalf`*.* Make sure you find *all the zeroes. Justify your answer.*

4. (EXTRA CREDIT) Define a `Maple` function g that, given a positive integer k yields the sum of the first k primes. What is k such that $g(k)\le 100,000$ but $g(k+1)>100,000$? You might find `sum` and `ithprime` helpful.

5. Find a polynomial of degree 3 passing through the points

   $$(-3.3,1),(-2,2),(0,3),(1.1,4)$$

   by two methods: First solve the equations (as in the book). Second, use the build-in `Maple` command. Check that the two solutions agree.

6. (a) Find two polynomials of degree 5 passing through the points

   $$(-3.3,1),(-2,2),(0,3),(1.1,4).$$

   (b) Plot the two polynomials and the points in the same graph.

   (c) Is it possible to find more polynomials of degree 5 passing through the 4 points? Why?

   (d) How many polynomials of degree 3 passing through the 4 points can you find?

7. Find a polynomial which passes through the points

   $$(-3,(-9)/4),(-2,(-3)/2),(-1,(-3)/4),(0,0),$$

   $$(1,3/4),(2,3/2),(3,9/4),(4,3),(5,15/4),(6,9/2),(7,5.35),(8,6).$$

   Then find a piecewise linear function passing through them.

8. Fit the points given in the previous problem to a line using the least square method. Plot the three results (the to of the previous problem, and the one you just obtained) and the points in the same graph. Compare the results. Which one do you think is best and why?

9. Fit the points $(-1.9, -4.7), (-0.8, 1.2), (0.1, 2.8), (1.4, -1.2), (1.8, -3.5)$ by means of a quadratic function $f(x) = ax^2 + bx + c$, using the least square method. First, do this step by step, as we did in class; then, use the built-in `Maple` command, described in the notes. Check that the two solutions agree.

10. Fit the set of points

$$(1.02, -4.30), (1.00, -2.12), (0.99, 0.52), (1.03, 2.51), (1.00, 3.34), (1.02, 5.30)$$

with a line, using the least square method we used in class. You will see that this is not a good fit. Think of a better way to do the fit and use `Maple` to do it. Explain in your solution why you think your better way is better.

11. Following the book, prove that if we describe the circle of center (a, b) and radius r using the parameters (a, b, k), with $k = a^2 + b^2 - r^2$, rather than the more natural parameters (a, b, r), then the error function $H(a, b, k) = E(a, b, \sqrt{a^2 + b^2 - k})$ is quadratic in a, b and k. What does this imply about the number of critical points?(Note: You do not need `Maple` in this exercise, altough you can use it as a word processor. If you're more confortable with paper, you can turn in a paper instead of a `Maple` worksheet)

12. (EXTRA CREDIT) With reference to the previous problem, show that, for $r > 0$, the transformation $(a, b, r) \mapsto (a, b, k)$ is a valid change of variables, that is, it is one-to-one. This should help you prove that $E(a, b, r)$ has only one "physical" critical point, which is a minimum, and is mapped, through the transformation, into the unique critical point of $H(a, b, k)$. (Note: You do not need `Maple` in this exercise, altough you can use it as a word processor.If you're more confortable with paper, you can turn in a paper instead of a `Maple` worksheet)

13. Fit the points $[[1, 1], [2, 3], [3, 1], [1, -1.1]]$ to a circle.

14. Following the on-line notes, find a set of 25 points which are "approximately" in a cubic and:

    (a) Find the cubic that best fit the points by the least squares method using the CurveFitting package.

    (b) Find the cubic that best fit the points by the least squares method without using the CurveFitting package.

    (c) (EXTRA CREDIT) Find the corresponding spline.

15. *[In this problem use* ***`Maple`*** *only as a word processor. If you're more confortable with paper, you can turn in a paper instead of a* ***`Maple`*** *worksheet.]* Let n points of the form (r_i, r_i^2), $i = 1, 2, \ldots, n$, be given. What is the quadratic function $f(x) = ax^2 + bx + c$ that best fits them? **Prove** your answer. Does it depend on the optimization method (least square or others)?