

The mathematical visions of Alicia Boole - fictional letters between Alicia Boole Stott and Lucy Everest Boole.

Moira Chas

Stage Note: Two actresses read the letters. The titles of the letters are not supposed to be read. The year and titles of the letters should be projected before and during the letters, stopping for the animations.

1 1894 Alicia to Lucy: The vision is complete

Dear Lucy,

I hope you recovered from your cold, and that you are taking better care of yourself. I suspect now that you have your project going, you can barely find time to perform the necessary acts of life like eating, sleeping and breathing. But please, once in a while, think of your older sister ordering you to stop and pay some attention to your health.

I have news that I can't wait to share with you: after weeks of being stuck, I have made progress in the understanding I have been searching for all these years and I am very close now to seeing the whole picture. I am aware, dear sister, that you do not share my passion for math, but you do know about that moment: the moment when the blurry ideas become clear and sharp, and the connections between the different aspects of whatever you are studying sound like a symphony. Everything fits into something of a nature that is beyond human. There is an enormous peace and joy that comes with understanding. Before this moment, the question was stinging inside me, creating discomfort, not letting me fill my lungs with air. Now, I do not seem to be able to stop talking about my happiness. But I do not want to abuse your kindness, so I ask: Would you care to read about my progress?

2 1894 Lucy to Alicia: Tell me about math

Stage Note: Lucy reads. Her voice shows evidence of a cold.

My dearest Alicia,

You can always write to me about the progress of your mathematical ideas. I am almost offended that you asked. My only request is that you start from the very beginning. Geometry has not visited my mind for quite some time.

I do try to take care of my health... once in a while. This job as lecturer is so demanding that I find it hard to find time for my research. Teaching is very draining. I suspect that I lose some of the respect of my students because I am a woman. One would think that female students would have less prejudice than their male counterparts, but this is far from the case. Sometimes, I feel like carrying a sign saying, "I am a Fellow of the Institute of Chemistry," but I am afraid that would not work either. The lack of justice in this situation makes me burn

with anger. I wish I had your strength, Alicia, and could carry on only with my inner life. But I need positive nods from the world around me.

Give my love to Walter and tell him that I said he should give you more time to work on math. Kiss the children for me.

3 1894 Alicia to Lucy: Alicia's vision

I like your request of starting it from the very beginning. It will help me organize my ideas.

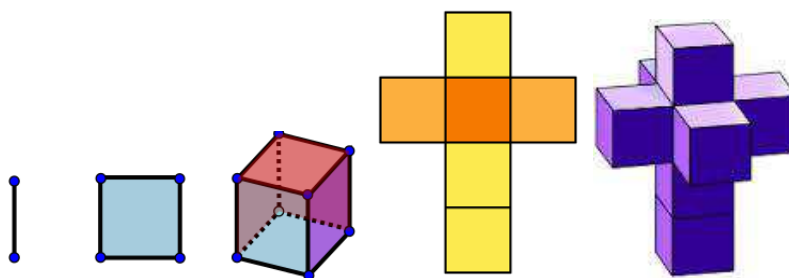


Figure 1: Segment, square, cube, net of a cube, net of a tesseract.

Stage Note: During this paragraph, (see Figure 1) animations should be projected.

- 1. one segment.*
- 2. four segments (consecutive, aligned) and then forming a square.*
- 3. a net of six squares on a plane, and then form a cube.*
- 4. a net of eight cubes on space with coloring indicating which pairs of sides should be glued.*

A segment is a one-dimensional shape.

Four congruent segments glued through the endpoints as in the figure below bound a square, which is a two-dimensional shape.

By gluing six squares in appropriate pairs of edges one obtains a cube, a three dimensional shape also with wonderful symmetries. It is easy to start with the shape below, a net of squares.

Our next task is to glue appropriate pairs of faces of eight cubes to obtain a four dimensional cube, that Charles calls the tesseract. Of course, this gluing can only occur in our mathematical mind and not in our three dimensional world; in the same way that a net of six squares in two dimensional plane has to be bent in a three dimensional space to form a cube. A net of eight cubes like the one below might help your mathematical mind.

The most remarkable aspect of the shapes I am interested in is their symmetries, the many ways one can rotate and flip them and still obtain the same shape. One natural question to ask is: can we find all the geometrical shapes that share these symmetries. In the plane, there are infinitely many: the regular polygons. In our three dimensional world, the shapes sharing these properties are the five platonic solids, those represented by the wooden models that Mother made for us to play with. They are at this very moment on my kitchen table. Mary showed some interest in them. Leonard is still too young to distinguish one from the other. I spent many happy hours of my childhood with the five little wooden models. You may remember that Mother left them with me when you all went to London. Playing with them made me feel closer to all of you.

What shapes with these properties exist in four dimensions? I have never stopped thinking about this since the moment Charles brought home the tesseract. It turns out that there are

exactly six of them. Once that this fact was clear in my mind, I worked hard trying to find ways of visualize them. But this is topic for a future letter.

Lucy, I wish Father were still with us so I could tell him about this. Would I be able to communicate with him? I wonder. I think in figures. I do not know how he thought.

4 1895 Lucy to Alicia: Your knowledge is not yours

My dearest Alicia,

It makes me deeply happy to see you absorbed with such passion in the pursuit of mathematics. As Madame du Chatelet wrote, “Passions are what one needs to ask God for, if one dares to ask for something.” Your thirst to understand math is a marvelous gift, and together with your persistence and your mathematical vision, it is producing something great. I am still amazed that you can flourish while far from peers and taking care of a family. I doubt I would have been able to do that. At the same time, I can appreciate how wonderful it is to have a mind of your own, molded by your own thoughts and ideas. This gave Father a unique vision, and now you are following a like path.

The trust I have in your intellect gives me absolute certainty that your results are correct. Now that you have arrived, now that you reached the top of that mountain after years of climbing, it is time to talk to the world. Your knowledge is not yours, dear sister, it belongs to humanity. All those hours you spent sitting at the kitchen table brought something to our universe. You must get out of your shell and complete the cycle.

If you want, I can talk to my colleagues at The London School of Medicine for Women. I know that some of them have connections in mathematics. Or maybe we should reach out to Charles?

I have good news as well: the article I wrote with Wyndham was finally accepted to be published in the Proceedings of the Royal Society of London. Our new project is making slow progress. I find it hard to be patient, but I tell myself that slow progress is better than no progress.

My relationship with S. is irrevocably broken. I do not want to talk about it, so please do not ask.

Stage Note: Lucy coughs

5 1895 Alicia to Lucy: The improbable coincidence

My dearest Lucy,

Today was the longest day of my life. Walter woke me up this morning with amazing news. He found an article in a journal of mathematics with figures that are almost identical to my polytopes. I had to go through my whole daily routine before I could sit and try to process all this. I am thinking as I am writing, wondering what I will do.

Let me give you the details. Last night Walter attended his monthly meeting of the “Puzzles’s Friends” group, in the house of a new member, who is a mathematician. On a desk, there was a magazine open, showing a figure very similar to the three dimensional sections of polytopes I draw incessantly. It was a joke of destiny, to put Walter, an actuary, in front of such an improbable coincidence. Of course, his first thought was that he was hallucinating. Incredulous, he took one of my figures out of his pocket (he always takes it with him and calls it the love letter) and compared them. My heart is racing as I write these words. The figures clearly represented the same object. I feel like somebody just gave me an infinite fortune and I do not know what to do with it.

My beloved Walter asked to borrow the journal for his wife. I wish I could have seen the face of the mathematician when confronted with such a strange request. Walter felt embarrassed and mumbled something about Father and this seemed to calm his guest. He probably thought that Father trained me in math. You know well how many times I wished this were true. I have the article open in front of my eyes as I write. I understand the figures, but the words and equations are completely meaningless to me. They make me feel utterly inadequate.

The author of the article is a Dutch mathematician, Pieter Hendrik Schoute. I considered writing to him, but I would not know where to start. Dear Lucy, even if you did not have a formal education, you developed your intellect by talking to your peers. How can I explain my mental images? I am not even sure if there is any value in these images. Why would I bother Dr. Schoute with them?

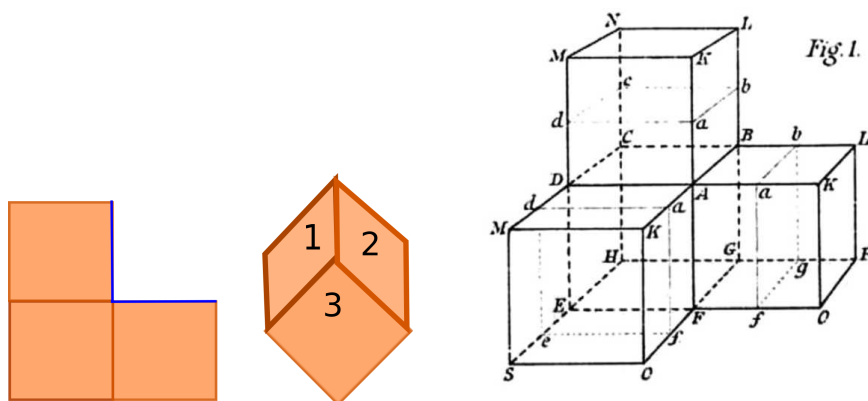


Figure 2: Vertex of a cube and Alicia drawing of the vertex of a tesseract by Alicia

Enough about me. Let me tell one one more little bit of math: recall the segment, square, cube and tesseract again. As you know, these are not the only figures I am studying, but they are a great example. Let's look at their vertices: A vertex of a square is touched by two segments. A vertex of a cube is touched by three squares. A vertex of a tesseract, as you expect, is touched by four cubes as the drawing below indicates. [Stage Note: See Figure 2.](#) In four dimensions, appropriate pairs of squares can be glued to form the vertex of a tesseract. This is a process I went through in my mind innumerable times. I glue one cube after another until the beautiful tesseract is formed.

Stage Note: During this paragraph, (see Figure 2 animations should be projected.)

- 1. a square with a vertex and the two segments emanating from it emphasized.*
- 2. a cube with a vertex and the three squares touching it emphasized.*
- 3. four cubes as in the figure below.*

I am including one of my drawings for you. You know how much I must love you to send you one of these. [Stage note: Figure 3 should be projected.](#)

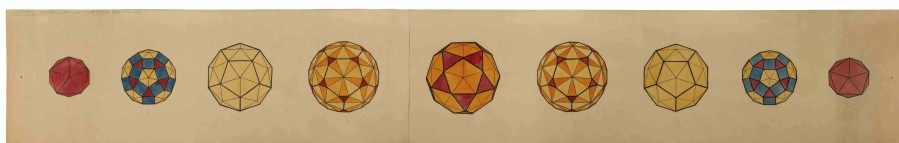


Figure 3: Parallel sections of the 120 cell

6 1895 Lucy to Alicia: If you do not write to Dr. Schoute I will

My dearest Alicia,

If you do not write to Dr. Schoute within a month, I will break into your house, find your drawings and send them myself. I mean it.

7 1895 Alicia to Lucy: The letter to Dr Schoute is in the mail

Dear Lucy,

The letter you are reading left the post office at the same time as a thick envelope addressed to Dr. Schoute. The envelope contained a short note talking about Father, the tesseract and a brief description of my ideas. I tried my best, but I am afraid that the scope of my sentences is very limited. I also included many of the photos that Walter took of the models. I trust the models much better than the words. Leaving the envelope at the post office gave me a wonderful sensation of relief. While walking home, I started working on my new project. As soon as I have something to tell, you will be the first to know. This happy moment would have not taken place without your help, and I need to thank you with all my heart.

I am sending you one of the photos Walter took in the hopes of piquing your curiosity. [Stage note: Figure 4 should be projected.](#)

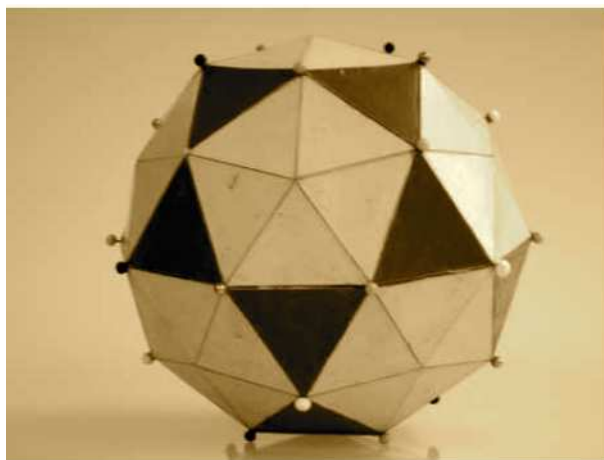


Figure 4: Model of the central diagonal section of the 600-cell

I am worried about your persistent cough. Please, pay a visit to your doctor. I beg you! You owe me that; I followed your advice regarding Dr Schoute. Besides, you are surrounded by physicians in your work. It should not be that hard to talk to one of them.

8 1896 Lucy to Alicia: I will see a doctor

Dear Alicia,

This is a short note to tell you that I will see a physician. Not one working in the college, because I want to keep my health separated from my work. You seem to forget I am a woman in a world of men. I have to weigh every step I take carefully. Many eyes are looking at me, trying to find something wrong.

On happier matters, know how much I applaud your decision to contact Dr. Schoute.

9 1896 Alicia to Lucy: My mind merged with Dr Schoute's

Dearest Lucy,

I am still waiting for news about your health. You are very busy, I know, but please do try to find a moment to write to me.

Summer has gone and we are all back to our old routine although things are getting easier now that the children are older and I do not have to wash diapers.

Those long hours discussing with Pieter Schoute in Hever changed me. I am still amazed and honored that he decided to come all the way from Groningen to work with me. In the beginning, our communication was worse than awful. There was a wall between us, formed by his limited English and, more importantly, my limited training in Math. But then, I brought my models, and he started to point at the different parts and related them with the equations of his papers. At some point, we managed to communicate so well that my conversation with him was like the dialogues I have had with myself during all these years. Time stopped existing, and if Aunt Ethel had not called us for dinner we would still be there, rotating the models and pointing to vertices. Even equations began to seem more natural although never as natural as the images. You were right, having a conversation with another mind takes the ideas to another level. I understood my own ideas better after discussing them with Pieter. More than discussing, it was like merging my mind with his.

I cannot find enough words to thank you again for your insistence that I should “get out of my shell”.

Pieter went back to the Netherlands with joint theorems to write. We have a complete, clear picture now and many problems to work on.

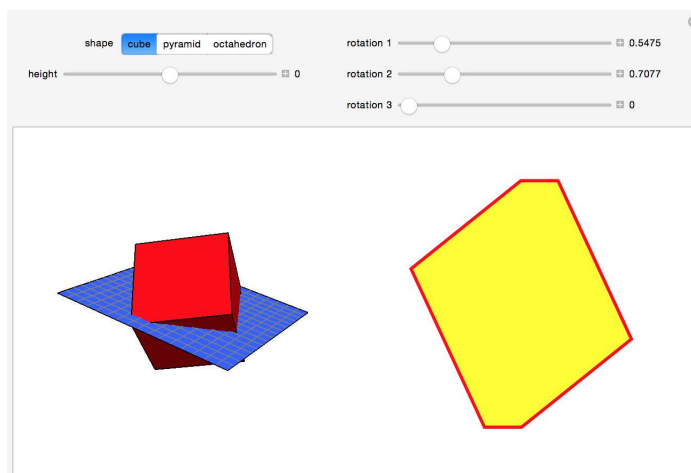


Figure 5: Sections of a cube

Stage Note: Animation of a cube intersected by a plane. If possible animation of sections of the tesseract.

Now, think of a cube and a plane floating in space. What shape can they form when they intersect? If the plane is parallel to one of the faces of the cube, they intersect in a square. Imagine now the plane is inclined and approaches the cube. First, they meet in a vertex, then in a triangle, then.... I will say no more. It is your job to imagine the rest. As Mother would say, construct a model and play with it.

We all missed you very much and are very sad that your health prevented you from coming. Ethel kept asking about you and the children complained that without Aunt Lucy nobody led them in carrying out interesting experiments. Pieter and I tried to drag them into math but with no real success. Leonard is especially interested in experiments. He claims he is going to be a doctor when he grows up, and he will take care of his Aunt Lucy.

10 1897 Lucy to Alicia: Tuberculosis

Dear Alicia,

There is no nice way to say what I have to say. I went to see Dr. Brown, and he says I have tuberculosis. My prospects are not great. I will not die tomorrow, but from now on, I will have to dedicate a great deal of time to treatments, and even so, I do not think I have many years left. Dr Brown did not say the latter, but I can read my labs and symptoms. I am not afraid of dying, Alicia. But, remember the verse of Gresset, "Pain is a century, death is a moment." I am not looking forward to my century of pain.

I enjoyed life very much, most hours of it. If I have to die, I will do it following my passions. What I hate the most is that I have to reduce the amount of hours I work. I discussed it in the college and was pleasantly surprised by their willingness to divide my position in half. Please do not share these news with the family yet, especially with Mother. I need time to digest all of this, and when I do, I will tell them myself. Try to minimize your sadness. We still have many things to share. Let's delay a conversation about my illness until it is inevitable. I needed to tell you, and now I need silence. I am aware of the strangeness of my request, but that does not make it less necessary for me.

Tell Leonard that I am sure he will be a marvelous physician.

11 1899 Alicia to Lucy: My work will be published

My dearest Lucy,

I will try my best not to burden you with the pain that your illness causes me and from this sentence on will follow your desire for silence. I will wait for your signal to talk. If there is anything, no matter what, that I can do to help, you know you can count on me.

I have news that you will like to read: Pieter arranged for my work on sections of polytopes to be published in *Verhandelingen der Koninklijke Akademie van Wetenschappen te Amsterdam* (I doubt I copied the name correctly). The cycle you encouraged me to close is now at the edge of closing. There is something very freeing about publishing. The topic is now out of my hands and leaves space in my mind for new problems. Speaking about these, Pieter just mailed me seventy pages full of his tiny handwriting. It will be very hard to understand, even after the long hours I spent talking to him. The language of my mind is made of figures.