Countability axioms

- A space is **first countable** if for each point p there is countable collection of neighborhood of p such that every neighborhood of p contains a member of the collection.
- A subset A of a space X is **dense** in X if Cl(A)=X.
- A space is **separable** if it has a countable dense subset.
- A space is **second countable** if there is a countable basis for its topology.
- A space is **Lindelof** if every open cover of X has a countable subcover.
- A space satisfies the **countable chain condition** if there does not exist an uncountable disjoint collection of open sets.

Separation axioms

(note: it what follows we assume x and y are distinct)

- T0: If x, y in X, there exists an open set U such that one of the following holds
  - x belongs to U and y does not belong to U
  - y belongs to U and x does not belong to U
- T1: If x, y in X, there exist a neighborhood of U and a neighborhood of y, V, such that x does not belong to V and y does not belong to U.
- T2: If x, y in X, there exists a neighborhood U of x and a neighborhood V of y such that U and V are disjoint.
- T3. If A is a closed set and b is a point not in A, there exist disjoint open sets U and V, containing A and b respectively.
- T4: If A and B are disjoint closed sets in X, there exist disjoint open sets U and V containing A and B respectively.
- T5: If A and B are separated then there exist disjoint open sets U and V containing A and B respectively. (**separated** means that Cl(A) does not intersect B and Cl(B) does not intersect A.).

- T2 1/2 : If x and y are in X, there exists a neighborhood U of x and a neighborhood V of y, such that Cl(U) and Cl(V) are disjoint.
- T3 1/2 : If A is a closed subset of X and x is a point not in A then there is a continuous function f:X -> [0,1], f(x)=1 and f(A)={0}.

- A **regular** space is a space that satisfies T0 and T3.
- A **normal** space is a space that satisfies T1 and T4.
- A **completely regular** space is a space that satisfies T0 and T3 1/2.
- A **completely normal space** is a space that satisfies T1 and T5.