Homework 12 MAT 515

Solve problems 1, 2, 3, 4, and 5.

1) Prove that a dilation preserves collinearity and angles.

2) If $\triangle ABC$ and $\triangle DEF$ are two triangles such that $\angle CAB \cong \angle FDE$ and $AB/AC = DE/DF$ then $\triangle ABC$ and $\triangle DEF$ are similar.

3) If $S$ is a similarity with ratio $s$ and $R$ is a similarity with ratio $r$ then $S \circ R$ is a similarity with ratio $r \cdot s$ and $S^{-1}$ is a similarity with ratio $1/s$.

4) Consider an isometry $T$. Determine the type of $T$ (rotation, reflection or glide reflection) in each of the following cases:
   (a) $T$ has no fixed points.
   (b) $T$ has exactly one fixed point.
   (c) The set of fixed points of $T$ is a line.

5) If two triangles $\triangle ABC$ and $\triangle DEF$ are similar, then there exist a positive real number $r$, such that $|AB| = r|DE|$, $|BC| = r|EF|$ and $|AC| = r|DF|$.

6) Determine all isometries of finite order, that is, all isometries $T$ such that $T^n = id$ for some positive integer $n$. You may use the fact that translations, reflections, rotations and glide reflections are the only isometries of the plane.