MY NAME IS: SOLAR ID:

# MAT 360 - Geometric Structures
# Practice Final

Dec , 2007

SHOW ALL WORK TO GET FULL CREDIT; A CORRECT ANSWER WITH INCORRECT OR NO JUSTIFICATION **will not get credit**.
CROSS OUT THE WORK YOU DO NOT WANT TO BE GRADED.

(1) Consider the following axiom system: **Undefined terms**: point, line, incidence.

**Axioms**

(i) There exist exactly four lines.
(ii) Any two distinct lines are incident with exactly one point.
(iii) Each point is incident with exactly two lines.

(a) Prove that there exists exactly six points.
(b) Prove that each line is incident with exactly three points.
(c) Prove that the axioms of Incidence Geometry are theorems of this axiom system.
(d) Is the system complete? Explain.
(e) Show the axioms are independent.
(f) Which, if any, of the parallel properties is a theorem of this axiom system? (Give a detailed answer for each parallel property).
(g) Find a model of this geometry.
(h) Is this axiom system categorical?
(i) Is this axiom system consistent?

(2) Choose one of the given interpretations of the undefined terms decide whether each of the proposed axioms hold (**Incidence Axioms, Betweeness Axiosm B1, B2 and B3 and B4** ). Study each of the parallel properties and indicate which one holds and which one does not hold.

(a) Let $S$ be a square in the Euclidean plane. **Interpretation**: Points are points in the interior of $S$ or in the edges of $S$. Lines are the segments with endpoints in the edges of $S$. For points $A$, $B$ and $C$, $A * B * C$ means $A$, $B$ and $C$ are collinear and $B$ is between $A$ and $C$ in the "Euclidean" sense.

(b) **Interpretation**: Points are points in the Euclidean plane. Lines are non-degenerate circles on the Euclidean. Incidence is set membership. A is between B and C if A, B and C are collinear.

(3) In neutral geometry, prove the hypothenuse-leg congruence condition: If two right triangles $\triangle ABC$ and $\triangle PQR$ have hypotenuses of equal length and a leg of one is congruent to a leg of the other, then $\triangle ABC$ is congruent to $\triangle PQR$. (Hint: Constructing an isosceles triangle may help.)

(4) Saccheri quadrilaterals are quadrilateras $\square ABCD$ such that angles $\sphericalangle DAB$ and $\sphericalangle ABC$ are right angles and $AD$ is congruent to $BC$.

(a) In neutral geometry, prove that Saccheri quadrilaterals exist. (You can prove existence by proving that it is possible to construct such a quadrilateral).

(b) In neutral geometry, consider a Saccheri quadrilateral $\square ABCD$. Let $E$ be the midpoint of $AD$ and let $F$ be the midpoint of $BC$. Prove that $\sphericalangle D$ is congruent to $\sphericalangle C$ and that $EC$ is congruent to $DF$.

(c) In Euclidean geometry: Do Saccheri quadrilaterals exist?. If your answer is yes, can you compute the sum of the interior angles?

(d) In hyperbolic geometry: Do Saccheri quadrilaterals exist? If your answer is yes, can you compute the sum of the interior angles?

(5) (a) Prove that in neutral geometry, Hilbert parallel postulate is equivalent to the converse of the alternate interior angle theorem.

(b) Does the equivalence in a. hold in hyperbolic geometry?

(6) (a) In neutral geometry, if $\triangle ABC$ is a triangle and $D$ is a point between $A$ and $B$ then the defect of $\triangle ABC$ is equal to the sum of the defects of $ACD$ and $BCD$. In symbols $\delta ABC = \delta ACD + \delta BCD$.

(b) Prove that in Euclidean geometry, the defect of every triangle is 0.

(c) In hyperbolic geometry, prove that for the triangles above, $\delta ABC > \delta BCD$ and $\delta ABC > \delta ACD$