3126. A Prime Number Conjecture
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we have
\[a = \tfrac{1}{2}\{p - c - \sqrt{(-3c^2 + 2cp + p^2 - 4q)}\},\]
\[b = \tfrac{1}{2}\{p - c + \sqrt{(-3c^2 + 2cp + p^2 - 4q)}\}.
\]
For these values to be real, it is necessary and sufficient that
\[3c^2 - 2cp - p^2 + 4q \leq 0,
\]
so that \( D \geq 0 \) and
\[c \in [\tfrac{1}{3}(p - 2\sqrt{D}), \tfrac{1}{3}(p + 2\sqrt{D})]. \quad \text{(iv)}
\]
Noting that \( a, b, c \) appear symmetrically in (iii), we deduce that
\[a, b, c \in [\tfrac{1}{3}(p - 2\sqrt{D}), \tfrac{1}{3}(p + 2\sqrt{D})]. \quad \text{(v)}
\]
The cubic equation with roots \( a, b, c \) has the form
\[t^3 - pt^2 + qt - abc = 0.
\]
The roots of the equation
\[
\frac{d}{dt}(t^3 - pt^2 + qt - abc) = 3t^2 - 2pt + q = 0
\]
are
\[t_1 = \tfrac{1}{3}(p - \sqrt{D}), \quad t_2 = \tfrac{1}{3}(p + \sqrt{D}).
\]
From Rolle’s theorem, the inequalities \( a < b < c \), and the relation (v) we obtain the required inequalities (ii).

This method may also be applied to more general cases.

Professor R. L. Goodstein, in his book *The uniform calculus and its applications* (Oxford, 1948, p. 356) has considered the system (i) when \( p = 2, q = 1 \). Goodstein’s procedure is more complicated than that used here.

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3126. A prime number conjecture

Let \( P_n \) denote the \( n \)th prime, and let a number \( N \) be said to be of type \( n \) if \( N \) is expressible as the sum or difference of two relatively prime numbers \( A_1, A_2 \) such that

(1) \( P_n \) is the largest prime which divides \( A_1 \) or \( A_2 \)

(2) every prime \( P_r, 1 \leq r \leq n \), is a factor of the product \( A_1 \cdot A_2 \).
I conjecture that every prime is of type \( n \) for some \( n \). The following table exhibits some numerical evidence which supports the conjecture. (The number 1 is counted as prime.)

Table giving types of Primes up to type 5.

<table>
<thead>
<tr>
<th>Prime Numbers</th>
<th>( P_2 = 3 )</th>
<th>( P_3 = 5 )</th>
<th>( P_4 = 7 )</th>
<th>( P_5 = 11 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>( 1 + 2 )</td>
<td>( 3.5 - 2^2 )</td>
<td>( 3.7 - 2.5 )</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>( 2 + 3 )</td>
<td>( 2^3 + 3 )</td>
<td>( 2^3 \cdot 3 )</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>( 2^2 + 3 )</td>
<td>( 2^3 + 3^2 )</td>
<td>( 2^4 \cdot 3 )</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>( 2^3 + 3 )</td>
<td>( 2^3 \cdot 3 )</td>
<td>( 2^3 \cdot 3 )</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>( 2^3 + 3^2 )</td>
<td>( 2^4 \cdot 3 )</td>
<td>( 2^5 \cdot 3 )</td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>( 2^3 + 3^2 )</td>
<td>( 2^4 \cdot 3 )</td>
<td>( 2^5 \cdot 3 )</td>
<td></td>
</tr>
<tr>
<td>19</td>
<td>( 2^3 + 3^2 )</td>
<td>( 2^4 \cdot 3 )</td>
<td>( 2^5 \cdot 3 )</td>
<td></td>
</tr>
<tr>
<td>23</td>
<td>( 2^3 \cdot 3 )</td>
<td>( 2^4 \cdot 3 )</td>
<td>( 2^5 \cdot 3 )</td>
<td></td>
</tr>
<tr>
<td>29</td>
<td>( 2^3 \cdot 3 )</td>
<td>( 2^4 \cdot 3 )</td>
<td>( 2^5 \cdot 3 )</td>
<td></td>
</tr>
<tr>
<td>31</td>
<td>( 2^3 \cdot 3 )</td>
<td>( 2^4 \cdot 3 )</td>
<td>( 2^5 \cdot 3 )</td>
<td></td>
</tr>
<tr>
<td>37</td>
<td>( 2^3 \cdot 3 + 1 )</td>
<td>( 2^4 \cdot 3 )</td>
<td>( 2^5 \cdot 3 )</td>
<td></td>
</tr>
<tr>
<td>41</td>
<td>( 2^3 \cdot 3 )</td>
<td>( 2^4 \cdot 3 )</td>
<td>( 2^5 \cdot 3 )</td>
<td></td>
</tr>
<tr>
<td>43</td>
<td>( 2^4 + 3^2 )</td>
<td>( 2^5 \cdot 3 )</td>
<td>( 2^5 \cdot 3 )</td>
<td></td>
</tr>
<tr>
<td>47</td>
<td>( 2^4 \cdot 3 )</td>
<td>( 2^5 \cdot 3 )</td>
<td>( 2^5 \cdot 3 )</td>
<td></td>
</tr>
</tbody>
</table>

The purpose of this note is to show that no other table is possible, despite the misgivings suffered by some people regarding the last two entries. For convenience we shall regard the arrangement of the \( p \) and \( q \) columns as fixed and write the entries in the third