<u>IMPORTANT NOTE: These are the answers. In the exam you need to add justifications</u>

Example (3.3-27)

Determine whether the following vectors form a basis of R^4

(1,1,1,1), (1,-1,1,-1),(1,2,4,8),(1,-2,4,-8)

Answer: Yes, (the matrix with these vectors as columns is invertible)

Exercise 1.2-30

Find the polynomial of degree 3 whose graph passes through the points (0,1),(1,0),(-1,0), (2,-15)

$$-2x^3 - x^2 + 2x + 1$$

Find the inverse of the rotation matrix.

$$\begin{vmatrix} \cos(a) & -\sin(a) \\ \sin(a) & \cos(a) \end{vmatrix} \quad \text{Answer=} \quad \begin{vmatrix} \cos(a) & \sin(a) \\ -\sin(a) & \cos(a) \end{vmatrix}$$

Let T be a clockwise rotation in R^2 by $\pi/2$ followed by an orthogonal projection onto the y axis.

1. Find the matrix of T.
2. Determine whether T is invertible
3. Find im(T) and ker(T)

Answer: It was given in class.

Find the inverse of the matrix and check your answer. Interpret your result geometrically.

$$\begin{vmatrix} a & b \\ b & -a \end{vmatrix}$$

Answer: The matrix is a reflection about a line L followed by a scaling by $(a^2 + b^2)^{1/2}$. The inverse is the same reflexion, followed by a scaling $(a^2 + b^2)^{-1/2}$

$$(a^2 + b^2)^{-1/2} \begin{vmatrix} a & b \\ b & -a \end{vmatrix}$$

For the matrix A below, find all the 2x2 matrices X that satisfy the equation $AX=I_2$.

$$\begin{vmatrix} 1 & 2 \\ 3 & 5 \end{vmatrix}$$

Answer: $$\begin{vmatrix} -5 & 2 \\ 3 & -1 \end{vmatrix}$$

(2.4-31)For which values of the constants a, b and c is the following matrix invertible?

$$\begin{vmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{vmatrix}$$

There are no values of a, b, c that make the matrix invertible

3.2-46 Find a basis of the kernel and image of the matrix.
Determine the dimensions of the kernel and image.
Determine the rank.
Justify your answers.

$$\begin{vmatrix} 1 & 2 & 0 & 3 & 5 \\ 0 & 0 & 1 & 4 & 6 \end{vmatrix}$$

Done in class

Example (3.3-31)

Let V be the subspace of R^4 defined by the equation $x_1 - x_2 + 2x_3 + 4x_4 = 0$

Find a linear transformation T from R^3 to R^4 such that $\ker(T)=\{0\}$, $\operatorname{im}(T)=V$.

Describe T by its matrix.

Note: The problem was first stated that T was from R to R^4 such. In this case, the answer is there are no transformations (why? Prove this!).

One answer for a transf. from R^3 to R^4 such is

$$\begin{vmatrix} 1 & 0 & 0 \\ 1 & 2 & 0 \\ 0 & -1 & 2 \\ 0 & 0 & -1 \end{vmatrix}$$

Give an example of a 5 x 4 matrix A with dim(ker A)=3. Compute dim(im A).

$$\begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{vmatrix}$$

The image has dimension 2.

Example (3.3-29)

Find a basis of the subspace of R^3 defined by the equation $2x_1+3x_2 + x_3 =0$

Answer: (3,-2,0), (0,1,-3)