MAT211 Lecture 15 Gram-Schmidt Process

- The Gram-Schmidt process
- QR Factorization

The Gram-Schmidt Process

EXAMPLE

Perform the Gram-Schimdt process on the sequence of vectors (1,1,1),(2,0,2),(-1,0,-1)

Theorem (QR Factorization)

Consider an n x n matrix M with linearly independent columns v_1, v_2,.. v_n.

Then there exists an n x n matrix Q whose columns u_1, u_2,.. u_n are orthonormal and an upper triangular matrix R with positive diagonal entries such that M = Q R.

The matrices Q and R are unique with the above properties. Moreover, $r_{11} = || v_1 ||$, $r_{jj} = || v_j^{\perp} ||$ for j=2..n, and $r_{ij} = u_i . v_j$ for i< j.

Theorem (QR Factorization Algorithm)

Consider an n x n matrix M with linearly independent columns v_1, v_2,.. v_n.

Then the columns q_1, q_2,.. q_n of Q and the columns of R can be computed in the following order

First col of R, first column of U

Second col of R, second col of U

and so on

EXAMPLE: Find the QR factorization of the matrix

$$\begin{vmatrix} 1 & 2 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 1 & 1 \end{vmatrix}$$