1. (5 points) Let \( f(x) = \frac{1}{x+1} \)
   (a) Find the average value of \( f \) in \([0, e^3 - 1]\).
   (b) Find \( c \) in \([0, e^3 - 1]\) such that \( f(c) \) is equal to the average value of \( f \) in \([0, e^3 - 1]\).

2. (6 points) A spring has a natural length of 5 cm. Suppose 2 joules of work is needed to stretch a spring from its natural length to a length of 30 cm.
   Find the length the spring will be stretched starting from a length of 15 cm if the work needed in this case is 10 joules.

3. (6 points) Verify that \( y = x \tan^{-1}(x) \) satisfies the differential equation
   \[ x(1 + x^2)y'' - 2y' + \frac{2}{x}y = 0 \]

4. (6 points) Solve the initial value problem
   \[ \frac{1}{\cos(x)} \frac{dy}{dx} = xy \] with \( y(0) = 3 \).

5. (6 points) Cobalt-60 has a half life of 5.24 years.
   (a) Find the mass that remains from a 100-mg sample after 20 years.
   (b) How long would it take for the mass to decay to 1 mg?

6. (5 points) A direction field for the differential equation \( y' = x^2 - y^2 \) is shown in Figure 1. Sketch the solution of the initial-value problem, \( y' = x^2 - y^2 \), \( y(0) = -1 \)

7. (5 points) Find the general solution of each of the following differential equations.
   (a) \( 2y'' - 4y' - 7y = 0 \)
   (b) \( y'' + 12y' + 36y = 0 \)

8. (6 points) Solve \( y'' - 2y' - 3y = 0 \) with \( y(0) = 3 \), \( y'(0) = -3 \).
Figure 1: Graph for problem 6