NAME: SOLAR ID:
RECITATION: LECTURE:

Problem	1	2	3	4	Total
Score					
Total Score	32	20	20	20	92

MAT 132 - Calculus II, Midterm 1 - Exam 1

March 2nd, 2011

(1) SHOW ALL WORK AND EXPLAIN REASONING WHENEVER POSSIBLE TO GET FULL CREDIT; A CORRECT ANSWER WITH INCORRECT OR NO JUSTIFICATION **will not get credit**.

(2) YOU HAVE 90 MINUTES TO COMPLETE THIS EXAM.

(3) YOU MAY NOT USE ANY BOOK, NOTES, CALCULATORS. OR ELECTRONIC DEVICES.

(4) CROSS OUT THE WORK YOU DO NOT WANT TO BE GRADED.

(5) SQUARE OR HIGHLIGHT YOUR FINAL ANSWERS.

Table of Integrals

$$\int x^n dx = \frac{1}{n+1}x^{n+1}, n \neq -1.+C \qquad \int e^x dx = e^x + C \qquad \int \sec x \tan x dx = \sec x + C$$

$$\int \frac{1}{x} dx = \ln x + C \qquad \int \sin x dx = -\cos x + C \qquad \int \csc^2 x dx = -\cot x + C$$

$$\int \cos x dx = \sin x + C$$

$$\int \frac{1}{1+x^2} dx = \arctan x + C \qquad \int \sec^2 x dx = \tan x + C \qquad \int \csc x \cot x dx = \csc x + C$$

(1) Evaluate each of the following indefinite integrals. Each one is worth 8 points.

(a) $\int \ln(1+x^2)$ dx.

Let $u = \ln(1+x^2)$, $dv = dx$

then $du = \dfrac{2x}{1+x^2}$, $v = x$

and $\int \ln(1+x^2)\,dx$

$$= \int u\,dv$$

$$= uv - \int v\,du$$

$$= x\ln(1+x^2) - \int \frac{2x^2}{1+x^2}\,dx$$

$$= x\ln(1+x^2) - \int \left(2 - \frac{2}{1+x^2}\right) dx$$

$$= x\ln(1+x^2) - 2x + 2\arctan x + C$$

#.

(b) $\int \sin^4(x)$ dx.

The double angle formula

$$\sin^2 x = \frac{1-\cos 2x}{2}$$

$$\cos^2 x = \frac{1+\cos 2x}{2}$$

gives $\int \sin^4 x\,dx$

$$= \int (\sin^2 x)^2\,dx$$

$$= \int \left(\frac{1-\cos 2x}{2}\right)^2 dx$$

$$= \frac{1}{4}\int (1 - 2\cos 2x + \cos^2 2x)\,dx$$

$$= \frac{1}{4}\int \left(1 - 2\cos 2x + \frac{1+\cos 4x}{2}\right) dx$$

$$= \frac{1}{4}\int \left(\frac{3}{2} + 2\cos 2x + \frac{\cos 4x}{2}\right) dx$$

$$= \frac{3}{8}x + \frac{1}{4}\sin 2x + \frac{1}{8}\sin 4x + C$$

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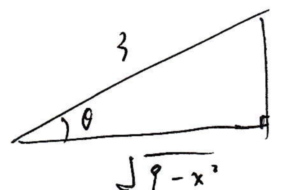
(c) $\int \frac{dx}{x^2\sqrt{9-x^2}}$.

Let $x = 3\sin\theta$, then $dx = 3\cos\theta\, d\theta$, $\sqrt{9-x^2} = 3\cos\theta$.

$$\Rightarrow \int \frac{dx}{x^2\sqrt{9-x^2}}$$

$$= \int \frac{3\cos\theta\, d\theta}{9\sin^2\theta \cdot 3\cos\theta}$$

$\sin\theta = \frac{x}{3}$



$$= \frac{1}{9}\int \csc^2\theta\, d\theta$$

$\cot\theta = \frac{\sqrt{9-x^2}}{x}$

$$= -\frac{1}{9}\cot\theta + C$$

$$= -\frac{1}{9}\,\frac{\sqrt{9-x^2}}{x} + C \quad \#.$$

(d) $\int \frac{dx}{x^2-x}$ dx

Let $\frac{1}{x^2-x} = \frac{1}{x(x-1)} = \frac{A}{x} + \frac{B}{x-1} = \frac{(A+B)x - A}{x(x-1)}$

then by comparing the coefficient,

we have $A = -1$, $B = 1$.

i.e. $\frac{1}{x^2-x} = \frac{1}{x-1} - \frac{1}{x}$.

$$\Rightarrow \int \frac{dx}{x^2-x} = \int \left(\frac{1}{x-1} - \frac{1}{x}\right) dx$$

$$= \ln|x-1| - \ln|x| + C$$

$$= \ln\left|\frac{x-1}{x}\right| + C \quad \#.$$

(2) For each of the following improper integrals:

(i) determine whether or not it converges.

(ii) Evaluate those that converge.

(a) $\int_1^{28} \frac{4}{(x-1)^{1/3}}$ dx.

Let $u = x-1$, then $\frac{1}{(x-1)^{1/3}}\,dx = u^{-1/3}\,du$

$$\Rightarrow \int_1^{28} \frac{4}{(x-1)^{1/3}}\,dx = 4 \lim_{t\to 0} \int_t^{27} u^{-1/3}\,du$$

$$= 4 \lim_{t\to 0} \frac{3}{2} u^{2/3} \Big|_t^{27}$$

$$= 6\left(\left(\sqrt[3]{27}\right)^2 - \lim_{t\to 0} t^{2/3}\right)$$

$$= 6 \cdot 9$$

$$= 54. \quad \text{converges.} \quad \#.$$

(b) $\int_3^{\infty} \frac{1}{x\ln^2(x)}$ dx.

Let $u = \ln x$, then $du = \frac{1}{x}\,dx$

$$\int_3^{\infty} \frac{1}{x \ln^2 x}\,dx$$

$$= \int_{\ln 3}^{\infty} u^{-2}\,du$$

$$= \lim_{t\to\infty} -u^{-1} \Big|_{\ln 3}^{t}$$

$$= \lim_{t\to\infty} \left(-\frac{1}{t} + \frac{1}{\ln 3}\right)$$

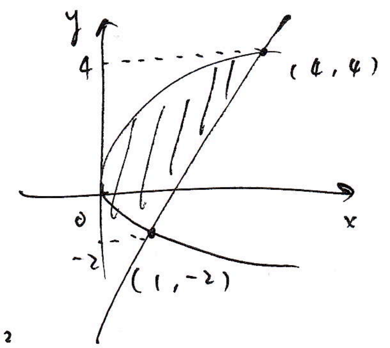
$$= \frac{1}{\ln 3}. \quad \text{converges.} \quad \#.$$

(3) Let R denote the region in the plane which is bounded by the curves $y^2 = 4x$ and $y = 2x - 4$.

(a) Express the area of R as a definite integral.

(b) Evaluate the definite integral of part (a).

The curves intersect at $(1, -2)$ and $(4, 4)$.



Rewrite the curves as functions of y's :

$$\gamma_1 : y = 2x - 4 \Rightarrow x = f_1(y) = \frac{y+4}{2}$$

$$\gamma_2 : y^2 = 4x \Rightarrow x = f_2(y) = \frac{y^2}{4}.$$

(a). Integrate with respect to y :

$$R = \int_{-2}^{4} \left(f_1(y) - f_2(y) \right) dy$$

$$= \int_{-2}^{4} \left(\frac{y+4}{2} - \frac{y^2}{4} \right) dy$$

(b)

$$= \left(\frac{y^2}{4} + 2y - \frac{y^3}{12} \right) \Bigg|_{-2}^{4}$$

$$= 9.$$

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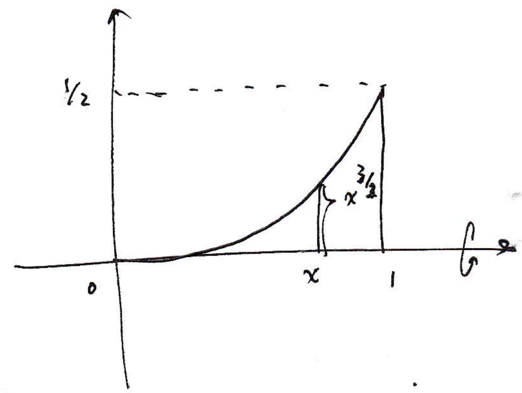
(4) Consider the region R bounded by $y = 2x^{3/2}$, $x = 1$ and the x-axis.

(a) Find the volume of the solid obtained by rotating R about the x-axis.

(b) Find the volume of the solid obtained by rotating R about the line $y = -2$.

(a) (Using the disk method)



The area of the cross-section disk at x is

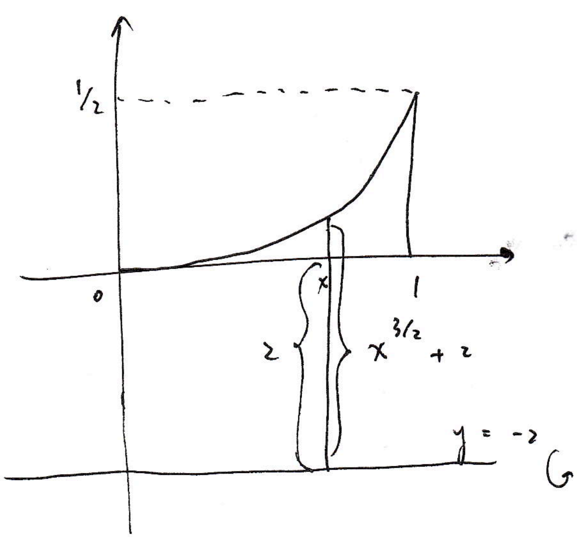
$$A(x) = \pi \left(x^{3/2}\right)^2 = \pi x^3$$

The volume of this solid lying between $x = 0$ and $x = 1$ is

$$V = \int_0^1 A(x)\,dx = \int_0^1 \pi x^3\,dx = \pi \frac{x^4}{4}\Big|_0^1 = \frac{\pi}{4}$$

(b) (Using the washer method)

The area of the cross-section washer at x is



$$A(x) = \pi\left[(x^{3/2}+2)^2 - 2^2\right]$$

$$= \pi\left(x^3 + 4x^{3/2}\right)$$

So the volume is

$$V = \int_0^1 A(x)\,dx = \pi \int_0^1 \left(x^3 + 4x^{3/2}\right) dx = \pi\left(\frac{x^4}{4} + 4\cdot\frac{2}{5}\cdot x^{5/2}\right)$$

$$= \frac{37\pi}{20}$$

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