MAT 132 - Calculus II, Practice Problems for Midterm 2

October 29th, 2014

(1) Show all work and explain reasoning whenever possible to get full credit; a correct answer with incorrect or no justification will not get credit.

(2) You have 90 minutes to complete this exam.

(3) You may NOT use any book, notes, calculators, or electronic devices.

(4) Cross out the work you do not want to be graded.

(5) Square or highlight your final answers.

(1) The region bounded by the curves $y = x^2$ and $x = y^2$ is rotated about the $x$-axis. Set up an integral for the volume of the resulting solid by two different methods.

(2) Set up (but do not evaluate) an integral for the length of the curve $y = x^{3/2}$, for $x \in [0, 4]$.

(3) Set up integral for the length of and the area of the inside loop of the polar curve $r = 1 - 2 \cos(\theta)$.

(4) A tank on the shape of a sphere of radius 10 ft is full of oil weighting 50 lb/ft$^3$. How much work is done by pumping the oil through a hole in the top?

(5) If 6 J of work is needed to stretch a spring from 10 cm to 12 cm and another 10 J is needed to stretch it from 12 cm to 14 cm, what is the natural length of the spring? (Recall Hooke’s law: the force required to maintain a spring stretched $x$ units beyond its natural length is proportional to $x$).
(6) Determine whether the each of the series is convergent or divergent. If it is convergent, find its sum. (Justify your answers)

(a) \( \sum_{n=1}^{\infty} \frac{1}{\sqrt{n^2+1}} \)

(b) \( \sum_{n=1}^{\infty} \left( \frac{3}{n(n+3)} + \frac{1}{(-10)^n} \right) \)

(c) \( \sum_{n=1}^{\infty} \frac{n^2-4}{2n^2+3} \)

(d) \( \sum_{n=1}^{\infty} \frac{\ln(n)}{n} \)

(7) Represent the number \( 3.4\overline{15} = 3.4151515\ldots \) as quotient of integers.

(8) (a) Approximate the series \( \sum_{n=1}^{\infty} \frac{1}{n^2} \) by using the first 4 terms.

(b) Estimate the error of the approximation.

(c) Determine how many terms are required to ensure that the sum is accurate to within 0.0001.