Review exercises for midterm I

1. Find the limit
   a. \( \lim_{x \to 2} \frac{x + 3}{x - 2} \)
   b. \( \lim_{x \to \infty} e^{x^3} \)
   c. \( \lim_{x \to -\infty} e^{x^3} \)

2. If an arrow is shot upward on the moon with a velocity of 70m/s, its height (in meters) after t seconds is given by \( H(t) = 70t - 0.83t^2 \).
   a. Find the velocity of the arrow after one second.
   b. Find the velocity of the arrow when \( t = b \).
   c. When will the arrow hit the moon?
   d. With what velocity will the arrow hit the moon?

3. Sketch the graph of a function \( g \) for which \( g(0) = 1, g'(0) = 2, g'(1) = 0 \) and \( g'(2) = -2 \).

4. Each limit represents the derivative of some function \( f \) at some number \( a \). State \( f \) and in each case and find the limit.
   a. \( \lim_{h \to 0} \frac{(2 + h)^3}{h} - 8 \)
   b. \( \lim_{h \to 0} \frac{\tan(h + \pi)}{h} \)
   c. \( \lim_{h \to 0} \frac{\cos(\pi + h) + 1}{h} \)
   d. \( \lim_{h \to 0} \frac{e^{2+h} - e^2}{h} \)

5. Decide about each of the following statements whether is true or false.
   a. If a function is differentiable at a point \( c \), then it is continuous at \( c \).
   b. There is a function \( F \) on the interval \((0, 6)\) with the following properties
      i. \( F' \) exists and is continuous
      ii. \( F \) is decreasing on \((0, 1)\)
      iii. \( F \) is increasing on \((5, 6)\)
      iv. \( F'(x) \) is different from 0 for all \( x \) in \((0, 6)\).
   c. If a function is continuous at -2 then it is differentiable at -2
   d. If \( f \) and \( g \) are differentiable functions then \( (f - g)' = f' - g' \)

6. Find the derivative using the definition of derivative. State the domain of the function and the domain of the derivative.
a.  \[ f(x) = x + \sqrt{x} \]

b.  \[ g(t) = \frac{2t}{t + 3} \]

7. The graph of the derivative of a function \( G \) is shown.
   a. On what intervals \( G \) is increasing or decreasing?
   b. At which values of \( x \) does \( G \) have a local maximum or minimum?
   c. If it is know that \( G(1)=3 \), sketch a possible graph of \( G \).

8. The graph of \( f \) is given. Sketch the graph of \( f' \).
9. Sketch the graph of a function that satisfies all the given conditions.
   a. \( \lim_{{x \to 4}} f(x) = -\infty \)
   b. \( f(x) < 0 \) if \( x \neq 4 \)
   c. \( f(0) = 0 \)
   d. \( f(x) > 0 \) if \( x < 0 \) or \( x > 4 \)

10. Differentiate the functions
   a. \( f(x) = \sin(2x)e^{x^3} \)
   b. \( g(x) = \frac{\cos x}{3x^4 + 2x^3} \)
   c. \( h(x) = \ln(\sqrt{x} + 3) \)
   d. \( r(x) = \frac{\cos x}{3x^4 + 2x^3} \)
   e. \( s(x) = \cos^{-1}(x^2) \)
   f. \( n(x) = e^x \ln x \)

11. Differentiate the function.
   a. \( f(x) = (\cos^{-1}(x)) \)
   b. \( u(x) = (\ln x)^x \)
   c. \( v(x) = \frac{\sin^2 x \tan^4 x}{(x + 1)^2} \)
   d. \( w(x) = \sqrt{\frac{x^2 + 1}{x^2 + 1}} \)
   e. \( f(x) = e^{e^x} \)
   f. \( g(x) = \frac{x}{\sin x + \cos x} \)
   g. \( h(x) = x^{\sin x} \)
11. If $f$ is a differentiable function, find an expression for the derivative of each of the following functions.
   a. $y=x^3f(x)^2$
   b. $y=x^2/f(x)$
   c. $y=(1+\sin x)f(x)/(x+3)$
   d. $y=\ln|f(x)|$

12. Find an equation to the tangent line to the curve $y=3/(1+e^{-2x})$ at the point $(0, \frac{3}{2})$

13. On what intervals is the curve $y=x^3-3x+7$
   a. increasing?
   b. concave upward?

14. Find an equation to the tangent line to the curve $y^2=x^3(2-x)$ at the point $(1,1)$

15. Find $dy/dx$ if a) $\cos(x-y)=xe^x$ b) $x^3+x^2+4y^2=6$

16. Find the linearization of $f(x) = \sqrt[3]{4x+1}$ at $a=0$. State the corresponding linear approximation and use to give an approximate value for $\sqrt[3]{1.03}$

17. Find the points on the ellipse $x^2+2y^2=1$ where the tangent line has slope 1.

18. Find the tangent to the curve $x^2y+xy^2=3x$ at the point $(2,1)$.

19. Let $f$ be a function such that $f(1)=2$ and $f'(x) = \sqrt{x^2+3}$. Use a linear approximation to estimate the value of $f(0.99)$. 