MAT131 – Spring 2003 –
Midterm II Solution

Problem	1	2	3	4	5	6	Total
Score							
Max	21	12	8				

Use of calculators, books or notes is not allowed. Show the all steps you made to find the answers. Write carefully, points may be taken off for meaningless statements.

1. Differentiate.
   a. $f(x) = e^x \sin(6x)$ $f'(x) = e^x \sin(6x) + 6e^x \cos(6x)$

   b. $f(x) = \tan(x + 7)$ $f'(x) = \sec^2(x + 7)$

   c. $f(x) = \sqrt[3]{x^2 - 5x}$ $f'(x) = \dfrac{2x - 5}{3\sqrt[3]{(x^2 - 5x)^2}}$

   d. $f(x) = (x^6 - 5)^{120}$ $f'(x) = 720x^5(x^6 - 6)^{119}$

   e. $f(x) = \sin^{-1}(x^2)$ $f'(x) = \dfrac{2x}{\sqrt{1 - x^4}}$

   f. $f(x) = x^{\cos x}$ $f'(x) = x^{\cos x}\left[\dfrac{\cos x}{x} - \sin x \ln x\right]$

   g. $f(x) = \ln(\sqrt{x})$ $f'(x) = \dfrac{1}{2x}$

   h. $f(x) = \dfrac{x^2 - 1}{\sin x}$ $f'(x) = \dfrac{2x \sin x - \cos x(x^2 - 1)}{\sin^2 x}$

2. Given the curve $y^2 - x^2 = \frac{3}{4}$.
   a. Find dy/dx by implicit differentiation.
   b. Find an equation of the tangent line to the curve $y^2 - x^2 = \frac{3}{4}$ at the point $(-2, -\sqrt{19}/2)$.
   c. Find all the points on the curve where the slope of the tangent line is 1/2.

a. $y' = \frac{x}{y}$

b. The slope of the tangent line at $(-2, -\sqrt{19}/2)$ is $4/\sqrt{19}$

The tangent line is y= - 19/2 +4/ 19(x+2)

c. The points where the slope is 1/2 are (1/2,1) and (-1/2, -1)

3. Let f be a function such that f(3)=5 and whose derivative is known to be $f'(x) = \sqrt{x^2 + 7}$. Use linear approximation to find the value of f(3.01).

Answer: f'(3)=4

The linear approximation at x=3 is L(x)=5+4(x-3).

$f(3.01) \approx 5+4(3.01-3)=5.04$

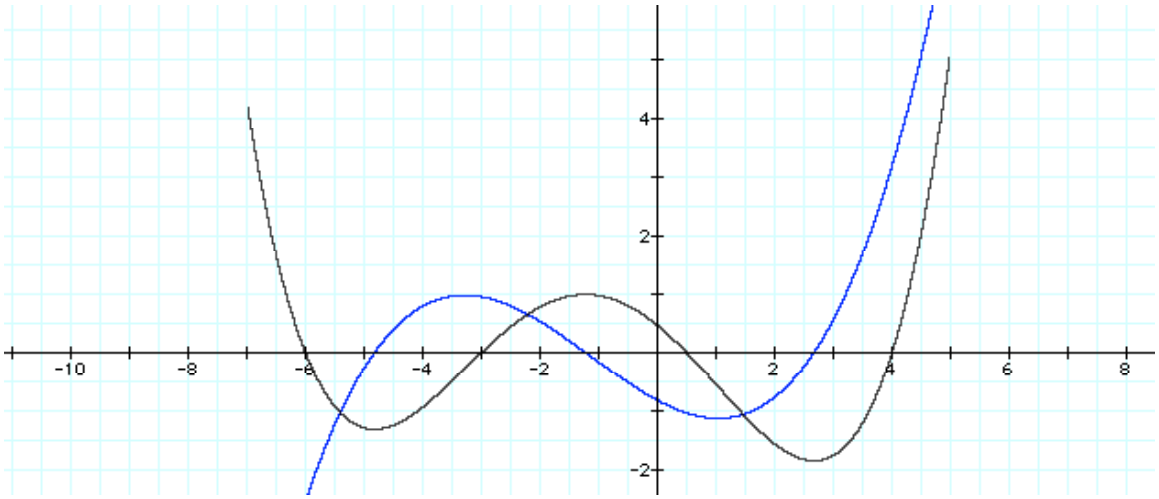
4. If f is a differentiable function, find an expression for the derivative of the following functions
   a. $h(x) = e^{2x} f(x)$ $h'(x) = e^{2x} f'(x) + 2e^{2x} f(x)$

   b. $h(x) = f(\ln x)$ $h'(x) = \frac{f'(\ln x)}{x}$

   c. $h(x) = [f(2x)]^3$ $h'(x) = 6[f(2x)]^2$

   d. $h(x) = \frac{\cos x + f(x)}{\sin x}$ $h'(x) = \frac{(-\sin x + f'(x))\sin x - (\cos x + f(x))\cos x}{\sin^2 x}$

5. The graph of f is given. Sketch the graph of the derivative of f, f' on the same set of coordinates.



6. Sketch the graph of a function with the following properties
   a. The domain of G is the interval [-3,4]
   b. G(0)=1, G'(0)=0
   c. G(3/2)=0, G'(3/2)=-1
   d. G'(x)>0 on [-3,0) and on (3,4]
   e. G''(x)<0 on (-1.5, 1.5)