Solution 0.1 *(Problem 1) Label the left and right vertices* l, r *Make an Adjacency matrix,* A*, with columns* a, b, l, r*:*

$$A = \begin{pmatrix} 0 & 2 & 2 & 1 \\ 2 & 0 & 1 & 2 \\ 2 & 1 & 0 & 2 \\ 1 & 2 & 2 & 0 \end{pmatrix}$$

Then:

$$A^4 = \begin{pmatrix} 177 & 136 & 136 & 176 \\ 136 & 177 & 176 & 136 \\ 136 & 176 & 177 & 136 \\ 176 & 136 & 136 & 177 \end{pmatrix}$$

1. *Thus the number of paths from* a *to* b *of length 4 is* 136
2. *The number of loops is the trace of* $A^4 = 4 \times 177 = 400 + 280 + 28 = 708$

Note that in class we oriented all edges.

Solution 0.2 *(problem 3)*

1. *the rank is the same as the dimension of the span of the column vectors. the first three columns, counting from the right are linearly independent so:*

   $$rank \geq 3$$

   On the other hand, The leftmost column vector is a linear combination of the middle two so:

   $$rank \leq 3$$

   Thus,

   $$rank = 3$$

2. *From the last part of this question, we know* $\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$ *is linearly dependent on the two given vectors.*

   The next two standard basis vectors, e_2, e_3 *are a natural next guess and do work as can be checked by taking, for example the determinant a the matrix having the given two* $\cup e_2, e_3$ *as columns.*

3. $i = 0$ *This follows by identifying each of the given polynomials with the given vectors in the previous part of the problem.*

Solution 0.3 *(Problem 5) Let's do this by cofactor expansion along the first column:*

$$(1)\begin{bmatrix} 6 & 7 & 8 \\ 0 & 9 & 10 \\ 0 & 11 & 12 \end{bmatrix} + (-5)\begin{bmatrix} 2 & 3 & 4 \\ 0 & 9 & 10 \\ 0 & 11 & 12 \end{bmatrix} + 0 - 0$$

now compute the 3×3*'s :*

$$(1)\left[(6)\left(9 \cdot 12 - 10 \cdot 11\right) - 0 + 0\right] + (-5)\left[(2)\left(9 \cdot 12 - 10 \cdot 11\right) - 0 + 0\right]$$

$$= (-4)(-2) = 8$$

Solution 0.4 *(Problem 7)*

1. *Let* Id_k *and* Id_{n-k} *be identity matrices. Then switching the first* k *for the last* $n-k$ *rows of a matrix is the same as multiplying by:*

$$\begin{pmatrix} 0 & Id_{n-k} \\ Id_k & 0 \end{pmatrix}$$

By induction on k *with* $j = n-k$ *fixed, you can show:*

$$det \begin{pmatrix} 0 & Id_{n-k} \\ Id_k & 0 \end{pmatrix} = (-1)^{(n-k)}$$

2.

$$rank(A) \leq rank(B) \leq 5$$

$\Rightarrow$ *A does not have full rank (would need rank* $= 10$*)* $\Rightarrow$ *A is singular* $\Leftrightarrow$ *det* $A = 0$

Solution 0.5 *(Problem 9)*

1.
   - $3:10$ *positive*
   - $6:05$ *positive*
   - $7:10$ *negative*
   - $12:00$ *1-dimensional.*
2. *negatively oriented. Area is 7.*

Solution 0.6 *(Problem 11) This follows from showing*

$$Im(A+B) \subset Im(A) + Im(B)$$