MAT303 Spring 2009

Practice Final

The actual final will consist of twelve problems with no more then two subproblems

You will be allowed to use calculators

Problem 1

Some of the given differential equations are separable and some are not. Solve those that are separable.

i $(1+x)ydx + xdy = 0$

ii $y' = y^{1/2}$

iii $y' + xy = 3$

iv $xy' - y\ln x = xy^2$

Problem 2

Some of the differential equations are linear and some are not. Determine those that are linear and give its integrating factor and solve them.

i $xy' + y = 3$

ii $xy' - y = 2x^2$

iii $y' - \frac{3}{x-1}y = (x-1)^4$

iv $y' + \frac{1}{\sin x}y - y^2 = 0$

v $xy' + y = x^5$

Problem 3

i) Is the equation exact? ii) If it is find the general solution. You may leave the answer in implicit form.

- $(3x - y)dx - (x + 3y)dy = 0$
- $(3x^2 - xy)dx - (x^2 + 3xy)dy = 0$, $x \neq 0$

Problem 4

Find the general solutions of differential equations (you may leave the answer in implicite form)

i $dy/dx = (x + y)/(2x - y)$

ii $dy/dx = xy + xy^4$

Problem 5

- The differential equation $dx/dt = \frac{1}{2}x(2-x)-h$ models a logistic population with harvesting at rate h. In the language of dynamics, one may say we are perturbing a logistic population by a constant h. So we usually want h to be small.

  i In terms of h, what are the equilibrium solutions?

  ii What are the stability of the solutions above? (Hint: Set $h = 0$, and then look at the stability there. This should tell you the stability of the solutions above.)

  iii What is the bifurcation point?

  iv Describe the stability of the bifurcation point. (Hint: Part ii)

  v For the problems below, set h to be the bifurcation point found in Part iv.

    a u is a solution with $u(2) = 5.5$. Compute $\lim_{t\to\infty} u(t)$.

    b u is a solution with $u(0) = 2$. Compute $\lim_{t\to\infty} u(t)$.

    c u is a solution with $u(0) = 1$. Using Eulers Method, approximate $u(2)$, using step size $\Delta x = 0.5$ to eight decimal places.

- Suppose that $-1 < a < 1$ is a constant parameter, and $y(t)$ satisfies the ODE

$$y^{'} = (a - y^2)(y - 2)$$

:

  - i Find the equilibria, sketch the phase line, and determine the stability of the equilibria in each of the following cases:
    - a $-1 < a < 0$;
    - b $a = 0$
    - c $0 < a < 1$.
  - ii Suppose that $y(t)$ is the solution of the ODE that satisfies the initial condition $y(0) = 0$. What is the behavior of $y(t)$ as $t \to \infty$ in each of the cases ia,ib,ic.

Problem 6

Compute the general solution of each nonhomogenenous equation by the Method of Undetermined coefficients

i $y^{''} + y = \sin x$

ii $y^{''} - y^{'} - 2y = 2xe^x + x^2$

iii $y^{''} - 5y^{'} + 4y = e^{2x}\cos x + e^{2x}\sin x$

Problem 7

Use the method of variation of parameter to solve the following initial value problem

i $y^{''} - y^{'} - 2y = t^2e^{2t}$, $y(0) = 0, y^{'}(0) = 1$

ii $y^{''} + y = -2\sin t, y(0) = 1, y^{'}(0) = 1$

Problem 8

Use the

- Eulers Method
- Improved Eulers Method

with $h = 0.2$ to solve the initial value problems on $0 \leq x \leq 1$

i $y' = 3x + 2y \ y(0) = 1$

ii $y' = xy \ y(0) = 1$

Problem 9

- Solve the second-order linear equation

$$y^{''} + 5y^{'} + 6y = 0$$

  i by using characteristic equation,

  ii by transforming it into a system of 2 first-order equations.

Problem 10

Find the general solution of the system $\frac{dx}{dt} = Ax$ using the method of ellimination where

$$A = \left(\begin{smallmatrix} 1 & -1 \\ 1 & 3 \end{smallmatrix}\right)$$

Problem 11

Find the general solution of the system $\frac{dx}{dt} = Ax$ using the method of eigenvalues where

i

$$A = \left(\begin{smallmatrix} -3 & 0 & -1 \\ 3 & 2 & 3 \\ 2 & 0 & 0 \end{smallmatrix}\right)$$

ii A is from Problem 10.

Problem 12

The motion of a mass on a spring can be described by the solution of the initial value problem $mu^{''} + cu^{'} + ku = F(t)$, $u(0) = u_0, u^{'}(0) = u_0^{'}$. A mass weighing 8 lb stretches a spring 6 in. The mass is attached to a viscous damper with a damping constant of 2 lb-sec/ft., and it is acted on by an external force of $\cos 3t$ lb. The mass is displaced 2 in. downward and released.

i Formulate the initial valued problem describing the motion of the mass.

ii Solve the initial valued problem using either the Method of Undetermined Coefficients or Variation of Parameters.

Problem 13

Find the general solution of the system

$$\begin{aligned} (D^2+1)x - D^2 y &= 2e^{-t} \\ (D^2-1)x + D^2 y &= 0 \end{aligned} \tag{1}$$

As usual $D = d/dt$