Practice Final Exam

Problem 3:

Is the equation exact?

If it is find the general solution. You may leave the answer in implicit form.

i) \((3x-y)dx-(x+3y)dy=0\)

Let \(M=(3x-y)\) \& \(N=-(x+3y)\)

Then \(My=-1\) \& \(Nx=-1\)

Since \(My \neq Nx\), the equation is \[
\boxed{\text{EXACT}}
\]

Integrate \(3x-y\) with respect to \(x\) to get \[
\frac{3x^2}{2}-xy+g(y),
\]

Let \(g(y)\) be some function of \(y\). 

If \(F(x,y)=\frac{3x^2}{2}-xy+g(y),\)

then \(F_y=-x-g'(y)=-x-3y.\)
or \( g'(y) = 3y \). Integrating this gives

\[ g(y) = \frac{3y^2}{2} + C \]

\( F(x,y) = \frac{3x^2}{2} - xy + \frac{3y^2}{2} \).

Thus, solutions to the exact differential equation are, in implicit form

\[ \frac{3x^2}{2} - xy + \frac{3y^2}{2} = C \]

ii) \( (3x^2 - xy) \, dx - (x^3 + 3xy) \, dy = 0 \) \( x \neq 0 \).

Let \( M = 3x^2 - xy \), \( N = -(x^3 + 3xy) \),

Then \( My = -x \) \& \( Nx = -2x - 3y \),

So the equation is \[ \text{NOT EXACT} \]