MAT203 Fall 2011

Practice Midterm II

The second midterm will contain 7 problems. You will not be allowed to use notes or calculators.


Problem 1 Determine which of the following equations define a cylinder and sketch its graph:

1. $x^2 + y^2 = 1 + z^2$.
2. $\ln(x) = y$.
3. $\cos(z) = y$.

Problem 2 Classify the surface defined by the following equations

1. $x^2 - 2x - 4 - y^2 - 4y - z = 0$
2. $3x^2 - 6x + 8 + 2y^2 + 8y - z^2 + 2z = 0$
3. $x^2 + 2x + 1 - 2y^2 + 4y - z^2 + 2z = 0$

Problem 3 Find equation of a surface of revolution obtained by rotation the curve given by equation $y = \ln(x)$ about

1. $x$-axis.
2. $y$-axis.

Problem 4 1. A surface in orthogonal coordinates is defined by equation

$$ x^2 - y^2 = 1. $$

Find its equation in cylindrical and spherical coordinates.
2. A surface in spherical coordinates is given by
\[ \rho \sin \phi \cos \theta + \rho \sin \phi \sin \theta - \rho \cos \phi = 1. \]
Find its equation in orthogonal system.

**Problem 5** Sketch the curve represented by vector-valued function
\[ r(t) = (2 - t)i + (\sqrt{t} + 1)j \]

**Problem 6** An acceleration function of an object satisfies
\[ a(t) = \sin ti + \cos 2t j + \cos(t + \pi/4)k \]
Find the position function \( r(t) \) if the initial velocity at time \( t = 0 \) is \( i - 2j + \sqrt{2}k \) and the initial position is \( 2i - 2j + 3k \).

**Problem 7** Find the unit tangent vector and the principal normal vector to the curve \( r(t) = ti - 2t^2 j - t^2 k \) at a point \( r(1) \).

**Problem 8** Find the arc length of the curve \( x = t^2, y = t^3 \) between (1, 1) and (4, 8).

**Problem 9** Sketch the level curves of the function
\[ f(x, y) = \begin{cases} 
  x + y^2 & \text{if } x > 0 \text{ and } y > 0 \\
  0 & \text{otherwise}
\end{cases} \]

**Problem 10** Identify limits that exist and evaluate them

1. \[ \lim_{(x,y)\rightarrow(0,0)} \frac{x + y}{x - y} \]
2. \[ \lim_{(x,y)\rightarrow(0,0)} \frac{xy}{x^2 + y^2} \]
3. \[ \lim_{(x,y)\rightarrow(0,0)} \frac{x^2y^2}{x^2 + y^2} \]

**Problem 11** Find the gradients of functions
1. \[ f(x, y) = \sin(\ln(x + y) \cos(xy)) \]

2. \[ g(x, y) = \frac{\sqrt{x + y + z}}{1 + x^2 + y^2 + z^2} \]

**Problem 12** Find \( \frac{\partial z}{\partial t} \) and \( \frac{\partial z}{\partial s} \) for \( z = e^{2r \sin(3\theta)}, r = st - t^2, \theta = \sqrt{s^2 + t^2} \)

**Problem 13** A function \( z = g(x, y) \) satisfies equation \( F(x, y, g(x, y)) = 0 \), where
\[ F(x, y, z) = x^2 + zy + y^2 + zx^2 + z^3 \]
find partial derivatives \( g_x, g_y \) as functions of \( x, y, z \).

**Problem 14**
1. Find formula for a normal vector to level curves of the function \( f(x, y) = x^2 + 3x + y - y^3 \).
2. Find critical (extreme) points of this function, determine their type.

**Problem 15** Find the equation of the tangent plane to the surface \( z = 3 + \cos(\pi xy) \) at the point \( (1, 1) \).

**Problem 16** Find the absolute maximum of the function
\[ f(x, y) = x^2 - 3xy + y^2 \]
in the region \( x^2 + y^2 \leq 1 \)

**Problem 17** Sketch the region of integration \( R \) and switch the order of integration in the following integrals
1. \[ \int_{0}^{4} \int_{0}^{\ln(x)} f(x, y) dy dx \]
2. \[ \int_{1}^{4} \int_{-\ln(x)}^{\ln(x)} f(x, y) dy dx \]
3. \[ \int_{2}^{3} \int_{2-y}^{1} f(x,y) \, dx \, dy \]

**Problem 18**

1. Evaluate \[ \int \int_{R} e^{-x-y} \, dx \, dy \]

where R is the region in the first quadrant in which \( x + y \leq 1 \)

2. Evaluate \[ \int_{0}^{1} \int_{x^{3}}^{2} \frac{dy}{1+y^{4}} \]

(Hint: change the order of integration first.)

**Problem 19**
The integral
\[ \int_{0}^{2} \int_{0}^{\sqrt{2x-x^{3}}} \sqrt{x^{2}+y^{2}} \, dy \, dx \]
is given in orthogonal coordinates. Change it to polar coordinates.

**Problem 20**

1. Find the volume of the solid that is below the surface \( z = 3x + 2y \) over the region \( R \) on the plane \( z = 0 \) bounded by the lines \( x = 0, y = 0 \) and \( x + 2y = 4 \) by evaluate a double integral.

2. Use polar coordinates to set up the integral for the volume of the solid inside the sphere \( x^{2} + y^{2} + z^{2} = 16 \) and outside the cylinder \( x^{2} + y^{2} = 4 \).

**Problem 21**
Find the area of the part of hyperbolic paraboloid \( z = y^{2} - x^{2} \) that lies between the cylinders \( x^{2} + y^{2} = 1 \) and \( x^{2} + y^{2} = 4 \).

**Problem 22**
Evaluate \[ \int \int_{V} 3xy \, dx \, dy \, dz \], where \( V \) is the solid between the \( xy \)-plane and the hyperbolic paraboloid \( z = xy \) for \( 0 \leq y \leq x, 0 \leq x \leq 1 \).