MAT203 Fall 2011

Practice Midterm II

The second midterm will contain 7 problems. You will not be allowed to use notes or calculators.

Sections covered: 11.6, 11.7, 12.1, 12.2, 12.3, 12.4, 12.5, 13.1, 13.2, 13.3, 13.4, 13.5, 13.6, 13.7, 13.7, 13.8, 14.1, 14.2, 14.3, 14.5, 14.6

Problem 1 Determine which of the following equations define a cylinder and sketch its graph:

1. $x^2 + y^2 = 1 + z^2$.
2. $\ln(x) = y$.
3. $\cos(z) = y$.

Problem 2 Classify the surface defined by the following equations

1. $x^2 - 2x - 4 - y^2 - 4y - z = 0$
2. $3x^2 - 6x + 8 + 2y^2 + 8y - z^2 + 2z = 0$
3. $x^2 + 2x + 1 - 2y^2 + 4y - z^2 + 2z = 0$

Problem 3 Find equation of a surface of revolution obtained by rotation the curve given by equation $y = \ln(x)$ about

1. x-axis.
2. y-axis.

Problem 4

1. A surface in orthogonal coordinates is defined by equation

$$x^2 - y^2 = 1.$$

Find its equation in cylindrical and spherical coordinates.

2. A surface in spherical coordinates is given by

$$\rho \sin\phi \cos\theta + \rho \sin\phi \sin\theta - \rho \cos\phi = 1.$$

Find its equation in orthogonal system.

Problem 5 Sketch the curve represented by vector-valued function $r(t) = (2 - t)i + (\sqrt{t} + 1)j$

Problem 6 An acceleration function of an object satisfies

$$a(t) = \sin ti + \cos 2tj + \cos(t + \pi/4)k$$

Find the position function $r(t)$ if the initial velocity at time $t = 0$ is $i - 2j + \sqrt{2}k$ and the initial position is $2i - 2j + 3k$.

Problem 7 Find the unit tangent vector and the principal normal vector to the curve $r(t) = ti - 2t^2 j - t^2 k$ at a point $r(1)$.

Problem 8 Find the arc length of the curve $x = t^2, y = t^3$ between $(1, 1)$ and $(4, 8)$.

Problem 9 Sketch the level curves of the function

$$f(x, y) = \begin{cases} x + y^2 & x > 0 \text{ and } y > 0 \\ 0 \text{ otherwise} & \end{cases}$$

Problem 10 Identify limits that exist and evaluate them

1. $$\lim_{(x,y)\to(0,0)} \frac{x + y}{x - y}$$

2. $$\lim_{(x,y)\to(0,0)} \frac{xy}{x^2 + y^2}$$

3. $$\lim_{(x,y)\to(0,0)} \frac{x^2 y^2}{x^2 + y^2}$$

Problem 11 Find the gradients of functions

1.

$$f(x,y) = \sin(\ln(x+y)\cos(xy))$$

2.

$$g(x,y) = \frac{\sqrt{x+y+z}}{1+x^2+y^2+z^2}$$

Problem 12 Find $\frac{\partial z}{\partial s}$ and $\frac{\partial z}{\partial t}$ for $z = e^{2r}\sin(3\theta)$, $r = st - t^2, \theta = \sqrt{s^2+t^2}$

Problem 13 A function $z = g(x,y)$ satisfies equation $F(x,y,g(x,y)) = 0$, where

$$F(x,y,z) = x^2 + zy + y^2 + zx^2 + z^3$$

find partial derivatives g_x, g_y as functions of x, y, z.

Problem 14

1. Find formula for a normal vector to level curves of the function $f(x,y) = x^2 + 3x + y - y^3$.
2. Find critical (extreme) points of this function, determine their type.

Problem 15 Find the equation of the tangent plane to the surface $z = 3 + \cos(\pi xy)$ at the point $(1, 1)$.

Problem 16 Find the absolute maximum of the function

$$f(x,y) = x^2 - 3xy + y^2$$

in the region $x^2 + y^2 \leq 1$

Problem 17 Sketch the region of integration R and switch the order of integration in the following integrals

1.

$$\int_0^4 \int_0^{y^2} f(x,y)dxdy$$

2.

$$\int_1^4 \int_{-\ln(x)}^{\ln(x)} f(x,y)dydx$$

3.
$$\int_2^3 \int_{2-y}^{\frac{1}{y}} f(x,y)dxdy$$

Problem 18 1. Evaluate
$$\iint_R e^{-x-y}dxdy$$
where R is the region in the first quadrant in which $x + y \le 1$

2. Evaluate
$$\int_0^8 \int_{x^{\frac{1}{3}}}^2 \frac{dydx}{1+y^4}$$
(Hint:change the order of integration first.)

Problem 19 The integral
$$\int_0^2 \int_0^{\sqrt{2x-x^2}} \sqrt{x^2+y^2}dydx$$
is given in orthogonal coordinates. Change it to polar coordinates.

Problem 20 1. Find the volume of the solid that is below the surface $z = 3x + 2y$ over the region R on the plane $z = 0$ bounded by the lines $x = 0, y = 0$ and $x + 2y = 4$ by evaluate a double integral.

2. Use polar coordinates to set up the integral for the volume of the solid inside the sphere $x^2 + y^2 + z^2 = 16$ and outside the cylinder $x^2 + y^2 = 4$.

Problem 21 Find the area of the part of hyperbolic paraboloid $z = y^2 - x^2$ that lies between the cylinders $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$.

Problem 22 Evaluate $\iiint_V 3xydxdydz$, where V is the solid between the xy-plane and the hyperbolic paraboloid $z = xy$ for $0 \le y \le x, 0 \le x \le 1$.