2) Let's say that \( g(x) = \ln x \), then \( f(x) = \ln x - 1 \) will take the graph of \( g(x) \) and shift it 1 unit downward.

Looking at the graph of \( f(x) \), we notice that we can't find the area of \( f(x) \), but we can certainly find the "difference in areas" for \([1,4]\) using left endpoints.

\[
L_6 = \sum_{i=1}^{6} f(x_{i-1}) \Delta x, \quad \Delta x = \frac{b-a}{n} = 0.5
\]

\[
L_6 = 0.5(f(1) + f(1.5) + f(2) + f(2.5) + f(3) + f(3.5))
\]

\[
L_6 = 0.5(-1.63372172) \approx -0.816861
\]

Riemann Sum represents: \((\text{Sum of the areas of the two rectangles that are "above" the x-axis}) - (\text{Sum of the areas of the four rectangles that are "below" the x-axis})\)
Recalling the "Midpoint Rule" on pg. 360, we have: (using \( n = 4 \))

\[
\int_1^5 x^2 e^{-x} \, dx \approx \Delta x \left[ f(x_i) + f(x_{i+1}) \right], \quad \Delta x = \frac{b-a}{n} = \frac{1}{2} \quad \text{and} \quad x_i = \frac{1}{2}(x_{i-1} + x_i)
\]

\[
\int_1^5 x^2 e^{-x} \, dx = f(1.5) + f(3.5) + f(4.5) + f(4.5), \quad \text{midpoints are: } x_i = 1.5, \, x_2 = 2.5, \, x_3 = 3.5, \, x_4 = 4.5
\]

\[
\int_1^5 x^2 e^{-x} \, dx \approx (1.5)^8 e^{-1.5} + (3.5)^3 e^{-3.5} + (3.5)^3 e^{-3.5} + (4.5)^3 e^{-4.5} \approx 1.6099
\]

**Conclusion:** Since \( f(x) \geq 0 \) for \( 1 \leq x \leq 5 \), where \( f(x) = x^2 e^{-x} \), the value 1.6099 is the "area" under the curve of \( f(x) = x^2 e^{-x} \) for \( 1 \leq x \leq 5 \).

18.) Using definition 3', \( \int_a^b f(x) \, dx = \lim_{n \to \infty} \sum_{i=1}^{n} f(x_i) \Delta x \), we have:

\[
\lim_{n \to \infty} \frac{5}{n+1} \sum_{i=1}^{n} \frac{e^{x_i}}{1+x_i} \Delta x = \int_1^5 \frac{e^x}{1+x} \, dx, \quad a=1 \text{ and } b=5.
\]

**Conclusion:** The definite integral of \( \frac{e^x}{1+x} \) from 1 (lower limit) to 5 (upper limit) is:

\[
\int_1^5 \frac{e^x}{1+x} \, dx = \lim_{n \to \infty} \sum_{i=1}^{n} \frac{e^{x_i}}{1+x_i} \Delta x
\]

20.) If we replace \( \sum \) by \( \int \), \( x_i^* \) by \( x \), and \( \Delta x \) by \( dx \), we are left with:

\[
\lim_{n \to \infty} \sum_{i=1}^{n} f(x_i) \Delta x = \int_a^b f(x) \, dx
\]

Using this methodology: \( f(x) = 4 - 3x^3 + 6x^5 \)

The definite integral of \( 4 - 3x^3 + 6x^5 \) from 0 (lower limit) to 2 (upper limit) is:

\[
\int_0^2 (4 - 3x^3 + 6x^5) \, dx = \lim_{n \to \infty} \sum_{i=1}^{\infty} (4 - 3x_i^3 + 6x_i^5) \Delta x
\]

28.) Using the same mentality as problems 18, 20:

\[
\int_a^b f(x) \, dx = \lim_{n \to \infty} \sum_{i=1}^{n} f(x_i) \Delta x, \quad \Delta x = \frac{b-a}{n} \quad \text{and} \quad x_i = a + i \Delta x
\]

\[
\int_1^{10} (x - 4 \ln x) \, dx = \lim_{n \to \infty} \sum_{i=1}^{n} (x_i - 4 \ln x_i) \Delta x, \quad \Delta x = \frac{10-1}{n} = \frac{9}{n} \quad \text{and} \quad x_i = 1 + i \Delta x
\]

\[
\int_1^{10} (x - 4 \ln x) \, dx = \lim_{n \to \infty} \sum_{i=1}^{n} \left[ \left(1 + \frac{9i}{n}\right) - 4 \ln \left(1 + \frac{9i}{n}\right) \right] \frac{9}{n}
\]

**definite integral** \( \text{Riemann Sum} \)
32) Using the knowledge that we absorbed Exs. 4a, 4b:

a) Find the area of this right, \( \triangle \) with base \( b = 2 \) and height \( h = 4 \):

\[
\text{Area} = \frac{1}{2}bh = \frac{1}{2} \cdot 2 \cdot 4 = 4 \rightarrow \text{Hence} \quad \int_0^2 g(x) \, dx = 4
\]

b) Find the area of this semicircle with radius \( r = 2 \) (radius is found by drawing a vertical line from \((4, 0)\) to \((4, 3)\))

\[
\text{Area} = 2 \cdot \frac{1}{2} \pi r^2 = \frac{1}{2} \pi r^2 = \pi \rightarrow \text{Hence} \quad \int_0^6 h(x) \, dx = \pi
\]

c) \[
\left[ \text{Area from} \ x=0 \ \text{to} \ x=2 \right] + \left[ \text{Area from} \ x=6 \ \text{to} \ x=7 \right] - \left[ \text{Area from} \ x=6 \ \text{to} \ x=7 \right] = \int_0^7 g(x) \, dx
\]

Area from \( x=6 \) to \( x=7 \):

\[
\text{Area of right } \triangle \text{ is :} \quad \frac{1}{2}bh = \frac{1}{2} \cdot 2(1) = 1 \rightarrow \\
\text{Hence,} \quad \int_6^7 g(x) \, dx = \frac{1}{2}
\]

Using 32a, 32b, we now have:

\[
\int_0^7 g(x) \, dx = 4 + \frac{1}{2} - 2\pi = \frac{9}{2} - 2\pi
\]

34) Using \( a^2 + b^2 = r^2 \) (center at origin):

\[
x^2 + y^2 = 4 \rightarrow y = \sqrt{4-x^2}, \text{ radius } r = 2 \text{ (Note: } \sqrt{ } \text{ sign is omitted since only } f(x) = \sqrt{4-x^2} \text{ is given)}
\]

Since \( y \) is non-negative, the graph looks like \( f(x) \geq 0 \)

The area under the curve "\( y = \sqrt{4-x^2} \)" from \(-2 \) (lower) to \( 2 \) (upper) is:

\[
\int_{-2}^{2} \sqrt{4-x^2} \, dx = 2 \cdot \frac{1}{2} \pi r^2 = \frac{1}{2} \pi (2)^2 = 2\pi
\]
42) Equation 5 says:

\[ \int_a^c f(x) \, dx + \int_c^b f(x) \, dx = \int_a^b f(x) \, dx \]

\[ \int_0^4 f(x) \, dx = \int_0^1 f(x) \, dx + \int_1^5 f(x) \, dx \]

\[ \int_1^5 f(x) \, dx = \int_1^5 f(x) \, dx - \int_1^4 f(x) \, dx \]

Hence, the area of \( f(x) \) from \( x=1 \) to \( x=4 \) is: 12 - 3.6 = 8.4

44) Graphing this "piecewise" function, we get:

\[ \int_0^5 f(x) \, dx = \int_0^3 f(x) \, dx + \int_3^5 f(x) \, dx \]

\[ \int_1^3 f(x) \, dx = \text{Area of rectangle}(A) = \int_0^3 3 \, dx \]

Using formula 1 on pg. 361: \( \int_0^3 3 \, dx = 3(3-0) = 9 \)

\[ \int_3^5 f(x) \, dx = \text{Area of rectangle}(A_2) + \text{Area of } \Delta \text{ (A_3)} \]

\[ = 2.5 + \frac{1}{2}(3)(2) = 8 \]

Therefore, \( \int_0^5 f(x) \, dx = 9 + 8 = 17 \)