MAT-125 H.W. #1 Solutions (Partial)

Ch. 5.1 #1-3, 5, 11, 15, 17-19

1-) Lower Estimate: Using left endpoints | Upper Estimate; Using right endpoints

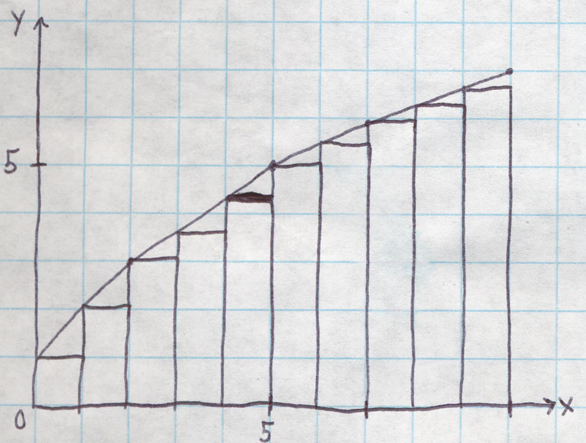
Lower Estimate (left endpoints)

a-)

$R_5 = \sum_{i=1}^{5} f(x_i)\Delta x \; ; \; \Delta x = \frac{b-a}{n} = \frac{10}{5} = 2$

$R_5 \approx 1\cdot2 + 3\cdot2 + 4.5\cdot2 + 5.5\cdot2 + 6.5\cdot2 \approx 41$

b-)

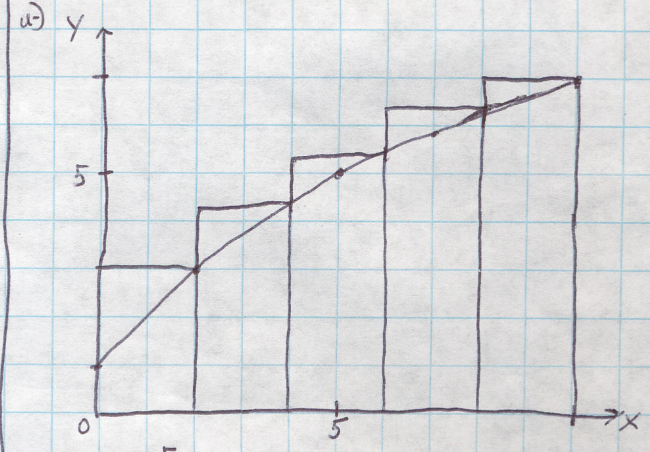


$L_{10} = \sum_{i=1}^{10} f(x_i)\Delta x \; ; \; \Delta x = \frac{b-a}{n} = \frac{10}{10} = 1$

$L_{10} \approx 1+2+3+3.5+4.5+5.0+5.5+\overset{6x}{6.2}+6.5 \approx \overset{43.2}{37.2}$

Upper Estimate (right endpoints)

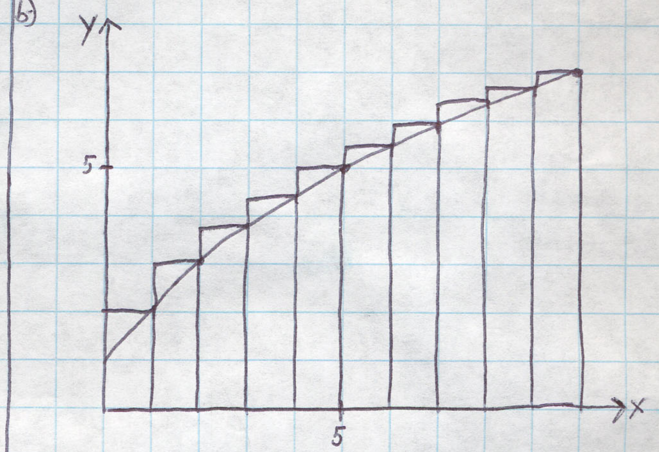
a-)



$R_5 = \sum_{i=1}^{5} f(x_i)\Delta x \; ; \; \Delta x = \frac{b-a}{n} = \frac{10}{5} = 2$

$R_5 \approx 3\cdot2+4.5\cdot2+5.5\cdot2+6.5\cdot2+7\cdot2 \approx 53$

b-)

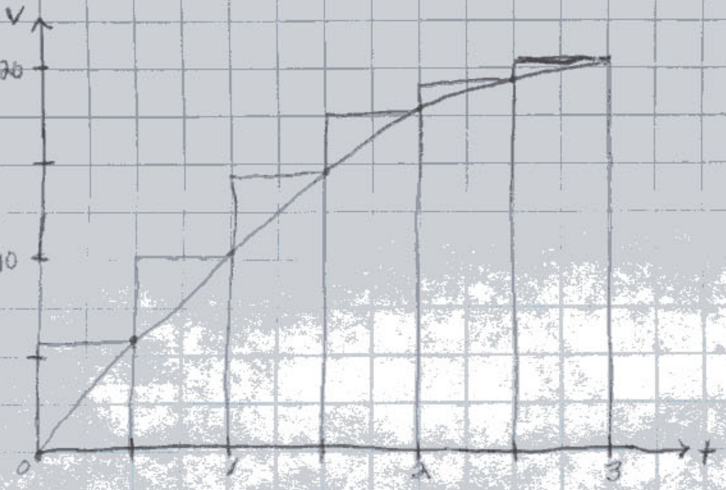


$R_{10} = \sum_{i=1}^{10} f(x_i)\Delta x \; ; \; \Delta x = 1$

$R_{10} \approx 2+3+3.5+4.5+5.0+5.5+\overset{6x}{6.2}+6.5+7 \approx 49.2$

Conclusion: "True" area is: $43.2 < A < 49.2$

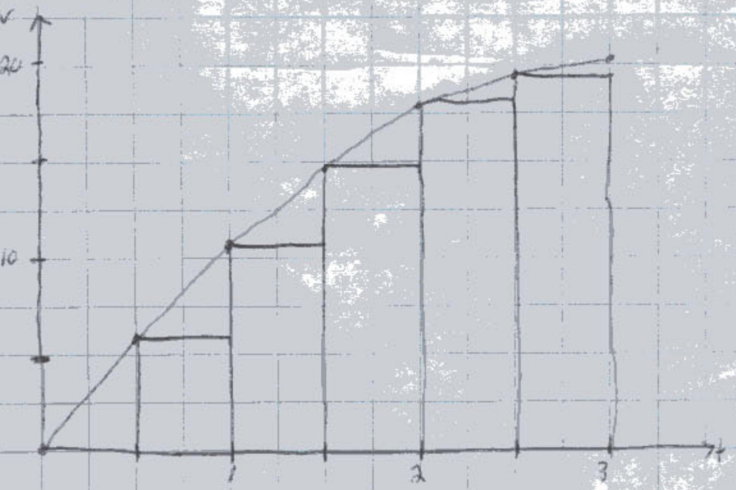
11-)



Upper Estimate:

$R_6 = 0.5(20.2 + 19.4 + 18.1 + 14.9 + 10.8 + 6.2)$

$R_6 \approx 44.8$



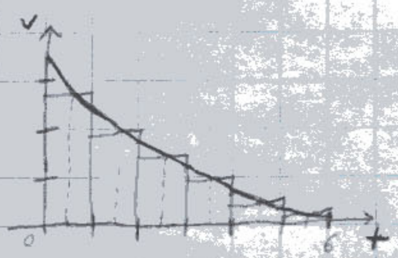
Lower Estimate

$L_6 = 0.5(6.2 + 10.8 + 14.9 + 18.1 + 19.4)$

$L_6 \approx 34.7$

$34.7\text{ft.} < A < 44.8\text{ft.}$

15-)



$M_6 \approx 55 + 40 + 30 + 17 + 10 + 3 \approx 155 \text{ ft.}$; $\Delta t = 1$

17-) $f(x) = \sqrt[4]{x}$, $1 \le x \le 16$

Using $a = 1$ and $b = 16$, we have:

$\Delta x = \frac{16-1}{n} = \frac{15}{n}$ and $x_n = a + n\Delta x$; $n = 1, 2, 3, \cdots$

Hence, $R_n = \frac{15}{n}\left(\sqrt[4]{1 + \frac{15}{n}} + \sqrt[4]{1 + \frac{30}{n}} + \cdots + \sqrt[4]{1 + \frac{15n}{n}}\right)$

Using Definition 2: $A = \lim_{n \to \infty} R_n = \lim_{n \to \infty} \sum_{i=1}^{n} \frac{15}{n}\left(\sqrt[4]{1 + \frac{15i}{n}}\right)$

18.) $f(x) = \frac{\ln x}{x}$, $3 \le x \le 10$

$\Delta x = \frac{b-a}{n} = \frac{7}{n}$ and $x_n = a + n\Delta x$;

Therefore, $R_n = \frac{7}{n}\left(\frac{\ln(3+\frac{7}{n})}{3+7/n} + \frac{\ln(3+\frac{14}{n})}{3+14/n} + \cdots + \frac{\ln(3+\frac{7n}{n})}{3+7n/n}\right)$

Using Definition 2: $A = \lim_{n\to\infty} R_n = \lim_{n\to\infty} \sum_{i=1}^{n} \frac{7}{n}\left(\frac{\ln(3+\frac{7i}{n})}{3+\frac{7i}{n}}\right)$

19.) We want to determine a region whose area is equal to $\lim_{n\to\infty} \sum_{i=1}^{\infty} \frac{\pi}{4n} \tan \underbrace{\frac{i\pi}{4n}}_{x_i}$

What do we do?

- Well, if we did problems 17, 18 correctly then we could use definition 2 and $x_i = a + i\Delta x$, to get:

$\Delta x = \frac{b-a}{n} = \frac{\pi}{4n} \rightarrow b-a = \frac{\pi}{4}$

$A = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$

$x_i = \overset{a=0}{0} + \frac{i\pi}{4n}$

$x_i = \frac{i\pi}{4n}$

- We clearly see that $f(x) = \tan x$ and that $a = 0$ ($\tan(0+\frac{i\pi}{4n}) = \tan\frac{i\pi}{4n}$) using $\boxed{x_i = a + i\Delta x} = i\Delta x$

Hence, the area we want is of $f(x) = \tan x$ from $[0, \frac{\pi}{4}]$.

One period of $y = \tan x$: