Problem 1

1. $\lim_{x \to \infty} \frac{x^3+2x+1}{2x^3-15x} = \lim_{x \to \infty} \frac{1+\frac{2}{x^2}+\frac{1}{x^3}}{2-\frac{15}{x^2}} = \frac{1}{2}$

2. $\lim_{x \to 2^-} \frac{x^2-2x-3}{x^2-5x+6} = \lim_{x \to 2^-} \frac{(x-3)(x+1)}{(x-3)(x-2)} = \lim_{x \to 2^-} \frac{x+1}{x-2} = -\infty$

3. $\lim_{x \to 3^+} \frac{x^2-2x-3}{x^2-5x+6} = \lim_{x \to 3^+} \frac{x+1}{x-2} = 4$

4. $\lim_{x \to \infty} \frac{1}{e^{(x^2)}+1} = 0$

5. $\lim_{x \to -1^+} \tan^{-1}\left(\frac{1-x}{1+x}\right) = +\frac{\pi}{2}$

6. $\lim_{x \to +\infty} \frac{\sin^{-1}\left(\frac{1-x}{1+x}\right)}{\cos^{-1}\left(\frac{1-x}{1+x}\right)} = \frac{\sin^{-1}(-1)}{\cos^{-1}(-1)} = \frac{-\frac{\pi}{2}}{\pi} = -\frac{1}{2}$

7. $\lim_{x \to 0} \frac{\sin(2x)^2}{x^2} = \lim_{x \to 0} \frac{2\sin(2x)\cos(2x)2}{2x} =$

$= \lim_{x \to 0} \frac{-2\sin^2(2x)2 + 2\cos^2(2x)2}{1} = 4$

Problem 2

1. $y = \frac{x^3+2x+1}{2x^3-32x} = \frac{x^3+2x+1}{2x(x-4)(x+4)}$

$\lim_{x \to \infty} \frac{x^3+2x+1}{2x^3-32x} = \frac{1}{2}$

horizontal asymptote: $y = \frac{1}{2}$

vertical asymptotes: $x=0,\ x=4,\ x=-4$

7. $\lim_{x \to 0} \frac{\sin(2x)^2}{x^2} = \cancel{\lim_{h \to 0}} \ \cancel{\sin(2}$

$$= \lim_{x \to 0} \frac{(2 \sin x \cos x)^2}{x^2}$$

$$= \lim_{x \to 0} \frac{4 \sin^2 x \cos^2 x}{x^2}$$

• near $x = 0$ $\sin x = x$

$$= \lim_{x \to 0} \frac{4 x^2 \cos^2 x}{x^2} = \lim_{x \to 0} 4 \cos^2 x = 4$$

2. $y = \dfrac{x^2-2x-3}{x^2-5x+6} = \dfrac{(x-3)(x+1)}{(x-3)(x-2)}$

$$\lim_{x\to\infty} \frac{x^2-2x-3}{x^2-5x+6} = 1$$

$$\lim_{x\to 3} \frac{x^2-2x-3}{x^2-5x+6} = \lim_{x\to 3} \frac{x+1}{x-2} = 4 \leftarrow \text{exists therefore no asymptote}$$

horizontal asymptote: $y=1$

vertical asymptote: $x=2$

3. $y = \dfrac{1}{e^{x^2}+1}$

$$\lim_{x\to\infty} \frac{1}{e^{x^2}+1} = 0$$

for vertical asymptotes $0 = e^{x^2}+1$

$-1 = e^{x^2}$ ← contradiction $e^{x^2} > 0$

horizontal asymptotes: $y=0$

vertical asymptotes: none

4. $y = \dfrac{\sin(2x)^2}{x^2}$

$$\lim_{x\to\infty} \frac{\sin(2x)^2}{x^2} = 0$$ ~~(see part 7 in Problem 1)~~

$$\lim_{x\to 0} \frac{\sin(2x^2)}{x^2} = 4$$ (see part 7 in Problem 1)

horizontal asymptotes: $y=0$

vertical asymptotes: none

Problem 3

1. $\frac{d}{dx}\left(\frac{1}{x^5}\right) = \frac{d}{dx}\left(x^{-5}\right) = -5x^{-6} = \frac{-5}{x^6}$

2. $\frac{d}{dx}(\cos x) = -\sin x$

3. $\frac{d}{dx}(e^x) = e^x$

4. $\frac{d}{dx}(x^2e^x) = 2xe^x + x^2e^x$

5. $\frac{d}{dx}(\tan(1+x^2)) = \sec^2(1+x^2)\,2x$

6. $\frac{d}{dx}[\sin^2 x] = 2\sin x\cos x$

7. $\frac{d}{dx}[\cos(xe^x)] = -\sin(xe^x)[e^x + xe^x]$

8. $\frac{d}{dx}\left[\frac{\cos(x^2)}{x^2}\right] = \frac{x^2(-\sin(x^2)2x) - 2x\cos(x^2)}{x^4} =$

$= \frac{-2x^2\sin(x^2) - 2\cos(x^2)}{x^3}$

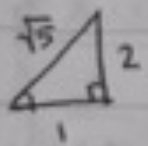
9. $\frac{d}{dx}\left[\sqrt{\sin\left(\frac{1}{x^2}\right)}\right] = \frac{\cos\left(\frac{1}{x^2}\right)\left(\frac{-2}{x^3}\right)}{2\sqrt{\sin\left(\frac{1}{x^2}\right)}} = -\frac{\cos\left(\frac{1}{x^2}\right)}{x^3\sqrt{\sin\left(\frac{1}{x^2}\right)}}$

10. $\frac{d}{dx}\left[\frac{1}{\sin^{-1}(x)}\right] = -\frac{1}{(\sin^{-1}(x))^2}\cdot\frac{1}{\sqrt{1-x^2}}$

11. $\frac{d}{dx}\left[\tan^{-1}(\sqrt{x})\right] = \frac{1}{(1+x)}\cdot\frac{1}{2\sqrt{x}}$

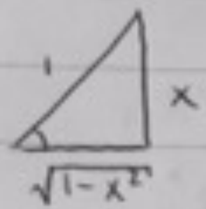
Problem 4

1. $\sin(\tan^{-1}(2)) = \frac{2\sqrt{5}}{5}$



2. $\cos(2\sin^{-1}(x)) = \cos^2[\sin^{-1}(x)] - \sin^2[\sin^{-1}(x)]$

$= (1-x^2) - x^2 = 1-2x^2$



Problem 5

1. $f(x) = x^4 + 7x + 3e^{x^3}$

$f'(x) = 4x^3 + 7 + 3e^{x^3}3x^2 = \boxed{4x^3 + 7 + 9e^{x^3}x^2}$

$f''(x) = 12x^2 + 9e^{x^3}2x + 9x^2e^{x^3}3x^2$

$= \boxed{12x^2 + 18e^{x^3}x + 27e^{x^3}x^4}$

2. $g(x) = \cos(x^4)$

$g'(x) = -\sin(x^4)\,4x^3 = \boxed{-4x^3\sin(x^4)}$

$g''(x) = -4x^3\cos(x^4)\,4x^3 - 4\sin(x^4)\,3x^2$

$= \boxed{-16x^6\cos(x^4) - 12x^2\sin(x^4)}$

Problem 6

$\frac{d}{dx}(xy + x^2y^3 = 10)$

$y + x\frac{dy}{dx} + 2xy^3 + x^2 3y^2\frac{dy}{dx} = 0$

$x\frac{dy}{dx} + x^2 3y^2\frac{dy}{dx} = -y - 2xy^3$

$f'(x) = \frac{dy}{dx} = \frac{-y-2xy^3}{x + x^2 3y^2}$ $f'(1) = -\frac{18}{13}$

Problem 5

$y = \sin(x) - 5x + 3 \qquad P(0,3)$

$y' = \cos(x) - 5$

at $(0,3)$ $\quad y' = \cos(0) - 5 = 1 - 5 = -4$

tangent line equation

$y - 3 = -4(x - 0)$

$y = -4x + 3$

$$\frac{d}{dx}\left(y + x\frac{dy}{dx} + 2xy^3 + x^2 3y^2\frac{dy}{dx} = 0\right)$$

$$\frac{dy}{dx} + \frac{dy}{dx} + x\frac{d^2y}{dx^2} + 2y^3 + 2x3y^2\frac{dy}{dx} + 2x3y^2\frac{dy}{dx} +$$

$$+ x^2 6y\left(\frac{dy}{dx}\right)^2 + x^2 3y^2\frac{d^2y}{dx^2} = 0$$

$$x\frac{d^2y}{dx^2} + x^2 3y^2\frac{d^2y}{dx^2} = -2\frac{dy}{dx} - 2y^3 - 12xy^2\frac{dy}{dx} - x^2 6y\left(\frac{dy}{dx}\right)^2$$

$$f''(x) = \frac{d^2y}{dx^2} = \frac{-2\frac{dy}{dx} - 2y^3 - 12xy^2\frac{dy}{dx} - x^2 6y\left(\frac{dy}{dx}\right)^2}{x + x^2 3y^2}$$

$$f''(1) = \frac{5108}{2197}$$

Problem 8

I - C

II - D

III - A

IV - B

Problem 9

1. $f(x) = \sin(x)\, g(x)$

$f'(x) = \cos(x)\, g(x) + \sin(x)\, g'(x)$

$f''(x) = -\sin(x)\, g(x) + \cos(x) g'(x) + \cos(x) g'(x) + \sin(x) g''(x)$

$f''\left(\frac{\pi}{3}\right) = -\sin\left(\frac{\pi}{3}\right) g\left(\frac{\pi}{3}\right) + 2\cos\left(\frac{\pi}{3}\right) g'\left(\frac{\pi}{3}\right) + \sin\left(\frac{\pi}{3}\right) g''\left(\frac{\pi}{3}\right)$

$= -\frac{\sqrt{3}}{2} \cdot 4 + 2 \cdot \frac{1}{2} \cdot 4 + \frac{\sqrt{3}}{2} \cdot 4 = 4$

2 $F(x)=G(H(x))$

$F'(x)= G'(H(x))\,H'(x)$

$F'(2)= G'(H(2))\,H'(2)= G'(3)\,9 = 9\cdot 9 = 81$