HW2

This is due Friday Feb. 15

1. Find all positive integers x such that $13|\ x^2 + 1$.

2. If $p$ is prime and $a^2 \equiv b^2 \text{ mod } p$, then $a \equiv b \text{ mod } p$ or $a \equiv -b \text{ mod } p$.

3. If $(a, m) = 1$ then
   a) Show that there is $b$ such that $ab \equiv 1 \text{ mod } m$. Also show that $(b, m) = 1$.

   Let $r_1, \ldots, r_s$ be a reduced system mod $p$ ($p$ is a prime number).
   
   b) Find $s$.
   
   c) Show that $r_1 \ldots r_s \equiv -1 \text{ mod } p$

4. Show that if $(a, b) = 1$ then $(a + b, a^2 - ab + b^2) = 1$ or $3$.
   
   (Hint: first use properties of G.C.D to simplify $(a + b, a^2 - ab + b^2) = 1$)

5. Using calculator find all solutions of $101x + 99y = 437$