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Biregular classification of Fano 3-folds and Fano manifolds of coindex 3

(Grassmann variety/vector bundle/K3 surface/homogeneous space/extremal ray)

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ABSTRACT **The Fano 3-folds and their higher dimensional analogues are classified over an arbitrary field $k \subset \mathbf{C}$ by applying the theory of vector bundles (in the case $B_2 = 1$) and the theory of extremal rays (in the case $B_2 \geq 2$). An n-dimensional smooth projective variety X over k is a Fano manifold if its first Chern class $c_1(X) \in H^2(X, \mathbf{Z})$ is positive in the sense of Kodaira [Kodaira, K. (1954) *Ann. Math.* 60, 28–48] (or ample). If $n = 3$ and $c_1(X)$ generates $H^2(X, \mathbf{Z})$, then either (*i*) X is a complete intersection in a Grassmann variety G with respect to a homogeneous vector bundle E on G: the rank of E is equal to $\mathrm{codim}_G X$ and X is isomorphic to the zero locus of a global section of E, (*ii*) X is a linear section of a 10-dimensional spinor variety $X_{12}^{10} \subset \mathbf{P}_k^{15}$, or (*iii*) X is isomorphic to a double cover of $\mathbf{P}_k^3$, a 3-dimensional quadric Q_k^3, or a quintic del Pezzo 3-fold $V_5 \subset \mathbf{P}_k^6$. If $n = 4$ and $c_1(X)$ is divisible by 2, then $X \otimes \mathbf{C}$ is isomorphic to (*a*) a complete intersection in a homogeneous space or its double cover, (*b*) a product of $\mathbf{P}^1$ and a Fano 3-fold, (*c*) the blow-up of $Q^4 \subset P^5$ along a line or along a conic, or (*d*) a $\mathbf{P}^1$-bundle compactifying a line bundle on $\mathbf{P}^3$ or on $Q^3 \subset \mathbf{P}^4$.**

This is a report on my study of Fano manifolds. The details will be published elsewhere.

For a projective variety $X \subset \mathbf{P}$, a complete intersection $X \cap P \subset P$ of $X \subset \mathbf{P}$ and linear subspace P of $\mathbf{P}$ is called a (*linear*) *section* of $X \subset \mathbf{P}$. The main subject of this report is the smooth projective variety $X \subset \mathbf{P}$ which satisfies the following equivalent conditions:

(*i*) $X \subset \mathbf{P}$ has a smooth curve section $C \subset P$ embedded by the canonical linear system $|K_C|$, and

(*ii*) $X \subset \mathbf{P}$ is linearly normal and the first Chern class $c_1(X)$ is cohomologous to $\dim X - 2$ times the cohomology class of hyperplane sections.

This type of projective variety was first considered by G. Fano (cf. ref. 1), who studied them by the method of double projection from lines and gave a birational classification over $\mathbf{C}$ in the 3-dimensional case. In this report, I shall give a biregular classification in an arbitrary dimension ≥ 3 over an arbitrary field of characteristic zero (*Theorem 2*). For this purpose the theory of vector bundles on a K3 surface (2) is applied in the case the second Betti number $B_2(X)$ is equal to one (*Section 2*). This method enables us to develop a canonical description of Fano 3-folds (of the first species) as subvarieties of Grassmann varieties (*Section 1*) and leads to an interesting relation between Fano 3-folds of genus 12 and classical projective geometry (*Theorem 5*).

A compact complex manifold X is a *Fano manifold* if $c_1(X)$ is positive in the sense of Kodaira (21). The greatest integer which divides $c_1(X)$ is called the index of X and the complete linear system $|H|$ with $rH \sim -K_X$ is called the fundamental linear system of X. The *coindex* of X is defined by $\dim X - r + 1$ (see refs. 3–5 for discussion of Fano manifolds of coindex ≤ 2). If X is a Fano n-fold of coindex 3, then the integer $g = \frac{1}{2}(H^n) + 1$ is called the *genus* and the fundamental linear system $|H|$ is of dimension $n + g - 1$. In the case $B_2(X) = 1$, the classification of Fano manifolds of coindex 3 is reduced to that of varieties with canonical curve sections under the following assumption:

Conjecture (*ES*) Every Fano manifold of coindex 3 has a smooth member in its fundamental linear system.

In the case $B_2(X) \geq 2$, the Fano manifolds of coindex 3 are classified by applying the theory of extremal rays (*Section 4*).

Section 1. Variety with a Canonical Curve Section

A surface with a canonical curve section is of type K3. A smooth projective variety X_{2g-2} of dimension $n \geq 3$ over k in a projective space $\mathbf{P}_k^{g+n-2}$ is an *F-manifold over k* if it satisfies the equivalent conditions *i* and *ii* above. An F-manifold $X \subset \mathbf{P}$ is *of the first species* if the Picard group Pic X is generated by the restriction of the tautological line bundle. I first give examples of F-manifolds of the first species.

For a vector space U, denote the Grassmann variety of s-dimensional subspaces of U by $G(s, U)$ and the projective space $G(1, U)$ by $\mathbf{P}^*(U)$. $G(s, U)$ is a projective variety in $\mathbf{P}^*(\bigwedge^s U)$ by the Plücker coordinate. The following two examples have been known classically.

($g = 8$): If U is a k-vector space of dimension 6, then the Grassmann variety $G(2, U) \subset \mathbf{P}^*(\bigwedge^2 U)$ is an 8-dimensional F-manifold of genus 8 over k.

($g = 6$): If U is of dimension 5, then a smooth hyperquadric section $X_{10}^6 \subset \mathbf{P}^{10}$ of the cone of $G(2, U) \subset \mathbf{P}^*(\bigwedge^2 U) \simeq \mathbf{P}^9$ is a 6-dimensional F-manifold of genus 6.

In the latter case, the F-manifold is a double cover of $G(2, U)$. Its isomorphism class depends on the multilinear form $F \in S^2(\bigwedge^2 U^\vee)$ on U which defines the branch locus. F-manifolds are obtained from Grassmann varieties for other values of g, too.

Example 1. Consider vector spaces and varieties over $\mathbf{C}$.

($g = 9$): Let U be a 6-dimensional vector space with a nondegenerate skew-symmetric bilinear form F. All the 3-dimensional subspaces W of U with $F(W, W) = 0$ form a smooth 6-dimensional subvariety X_{16}^6 of degree 16 in $G(3, U) \subset \mathbf{P}^*(\bigwedge^3 U)$.

($g = 10$): Let U be a 7-dimensional vector space with a nondegenerate (see *Remark 1*) skew-symmetric 4-linear form F. All the 5-dimensional subspaces W with $F(W, W, W, W) = 0$ form a smooth 5-dimensional subvariety X_{18}^5 of degree 18 in $G(5, U) \subset \mathbf{P}^*(\bigwedge^5 U)$.

($g = 12$): Let U be a 7-dimensional vector space and F_1, F_2, and F_3 be three linearly independent skew-symmetric bilinear forms on U. Let X be the subvariety of $G(3, U) \subset \mathbf{P}^*(\bigwedge^3 U)$ consisting of 3-dimensional subspaces W with $F_1(W, W) = F_2(W, W) = F_3(W, W) = 0$. If the subspace $F_1 \wedge U^\vee +$

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$F_2 \wedge U^\vee + F_3 \wedge U^\vee$ of $\bigwedge^3 U^\vee$ contains no vectors of the form $f_1 \wedge f_2 \wedge f_3 \neq 0$, $f_1, f_2, f_3 \in U^\vee$, then X is a smooth 3-dimensional subvariety X_{22}^3 of degree 22.

THEOREM 1. (i) *In each case of* Example 1, *the linear envelope of* $X_{2g-2}^{n(g)}$, g = 9, 10, 12, *and* n(g) = 15 − g, *is isomorphic to* $\mathbf{P}^{13}$ *and the projective variety* $X_{2g-2}^{n(g)} \subset \mathbf{P}^{13}$ *is an* n(g)-*dimensional F-manifold of the first species* (*over* **C**) *of genus* g.

(ii) *Let* U *be a 9-dimensional vector space with a nondegenerate symmetric bilinear form* F *and* S *be the space of spinors of* F (*cf. ref.* 6). *All the 4-dimensional subspaces* W *of* U *with* F(W, W) = 0 *form a smooth* 10-*dimensional subvariety* X_{12}^{10} *in* G(4, U), *called a spinor variety.* X_{12}^{10} *embedded in* $\mathbf{P}^*(S) \simeq \mathbf{P}^{15}$ *by the spinor coordinate is an F-manifold of the first species of genus* 7.

Remark 1. In each case $7 \leq g \leq 10$, the F-manifolds $X_{2g-2}^{n(g)}$ in *Theorem 1* are homogeneous spaces (7). This is less obvious in the case $g = 10$. If U is a 7-dimensional vector space, then $\bigwedge^3 U$ $(\simeq \bigwedge^4 U^\vee)$ contains an open GL(U)-orbit. If F is *nondegenerate*, i.e., belongs to the open orbit, then the stabilizer group G_F contains an algebraic group of type G_2 as its connected component (cf. ref. 8). The F-manifold X_{18}^5 is homogeneous with respect to G_F.

The isomorphism class of $X_{2g-2}^{n(g)}$ depends only on the vector space M spanned by a multilinear form $F(g \neq 12)$ or by multilinear forms F_1, F_2, and F_3 ($g = 12$). [$n(g)$ is equal to g for $g = 6$, 8 and $n(7) = 10$.] Hence we can denote it by $\sum_g(U, M)$. (We understand $M = 0$ in the case $g = 8$.) If U is a k-vector space and if M is defined over k, then so is the variety $\sum_g(U, M)$. In the case $g \neq 7$, we obtain an F-manifold $\sum_g(U, M) \subset \mathbf{P}_k^{g+n(g)-2}$ over k. In the case $g = 7$, an F-manifold $\sum_g(U, M) \subset \mathbf{P}_k^{15}$ over k is obtained if, in addition, the space of spinors S is defined over k.

THEOREM 2. *Let* X ⊂ **P** *be an F-manifold of the first species of genus* g ≥ 6 *over a field* k ⊂ **C**. *Then there exist a* k-*vector space* U *and a space* M *of multilinear forms on* U *such that* X ⊂ **P** *is isomorphic to a linear section of* $\sum_g(U, M) \subset \mathbf{P}_k^{g+n(g)-2}$.

Remark 2. (*i*) An F-manifold of genus ≤ 5 is a complete intersection of hypersurfaces.

(*ii*) For Fano 3-folds of genus 6 over **C**, the theorem is proved independently by Gushel (9).

The following is an easy consequence of the result of Iskovskih (10) on the anticanonical linear system.

PROPOSITION 1. *Let* X *be a Fano n-fold of coindex* 3 *with* $B_2(X) = 1$. *Under the assumption (ES), the fundamental linear system* |H| *is base point free and* X *satisfies one of the following:*

(i) |H| *is very ample and the image of* $\Phi_{|H|}$ *is an F-manifold of the first species, or*

(ii) *the morphism* $\Phi_{|H|}: X \to \mathbf{P}^{g+n-2}$ *is a finite morphism of degree* 2 *onto* $\mathbf{P}^n$ (g = 2) *or onto a hyperquadric* Q^n *in* $\mathbf{P}^{n+1}$ (g = 3).

The conjecture (ES) is a theorem of Shokurov (11) in the case dim $X = 3$ and has been proven by Wilson (12) for Fano 4-folds of index 2 and with $B_2 = 1$. Hence, *Proposition 1* and *Theorem 2* completely classify Fano 3-folds of the first species and Fano 4-folds of index 2 and with $B_2 = 1$.

Section 2. Classification by Means of Vector Bundles

An F-manifold has a smooth surface section of type K3.

THEOREM 3. *Let* S *be a K3 surface whose Picard group* Pic S *is generated by an ample line bundle* L *with* $(L^2) = 2g - 2$. *Then for every pair of positive integers* (r, s) *with* rs = g, *there exists a stable vector bundle* E *of rank* r, $c_1(E) = c_1(L)$, *and* $\chi(E) = r + s$ *which satisfies the following:*

(i) $H^1(S, E) = H^2(S, E) = 0$ *and* E *is generated by global sections,*

(ii) *every stable vector bundle* E′ *on* S *with the same rank and the same Chern class as* E *is isomorphic to* E (*cf. corollary 3.5 of ref* 2), *and*

(iii) *the natural linear mapping* $\lambda_r: \bigwedge^r H^0(S, E) \to H^0(S, \bigwedge^r E) \simeq H^0(S, L)$ *is surjective.*

For a vector space V, denote the Grassmann variety of r-dimensional quotient spaces of V by $G(V, r)$ and the projective space $G(V, 1)$ by $\mathbf{P}^*(V)$. By *Theorem 3*, we obtain a morphism $\Phi_{|E|}$ from S to the g-dimensional Grassmann variety $G(V, r)$ such that $\Phi_{|E|}^*\mathscr{E} \simeq E$ for every (r, s) with $rs = g$, where $V = H^0(S, E)$ and $\mathscr{E}$ is the universal quotient bundle on $G(V, r)$. We also obtain the embedding $\Psi = \mathbf{P}^*(\lambda_r)$: $\mathbf{P}^*(H^0(S, L)) \hookrightarrow \mathbf{P}^*(\bigwedge^r V)$. Let I_{r-1} be the kernel of the natural linear mapping $\lambda_{r-1}: \bigwedge^{r-1} V \to H^0(S, \bigwedge^{r-1} E)$. Then $P = \mathrm{Im}\,\Psi$ is a linear subspace of $\mathbf{P} = \mathbf{P}^*(\bigwedge^r V / I_{r-1} \wedge V) \subset \mathbf{P}^*(\bigwedge^r V)$. We obtain the following commutative diagram:

$$\begin{array}{ccc} \Phi_{|E|}: S & \to & G(V, r) \cap \mathbf{P} \subset G(V, r) \\ \cap & & \cap \\ \Psi: \mathbf{P}^*(H^0(L)) & \to & \mathbf{P} \subset \mathbf{P}^*(\bigwedge^r V) \end{array}$$

THEOREM 4. *Let* S, L, g, *and* E *be as in* Theorem 3.

(*i*) *If* (r, s) = (2, 4), (3, 3), (5, 2), *or* (3, 4) *and* g = rs, *then the intersection* G(V, r) ∩ **P** ⊂ **P** *is an* n(g)-*dimensional F-manifold* $\sum_g$ *and* $\Phi_{|E|}(S) \subset \mathbf{P}$ *is its linear section.*

(ii) *If* (r, s) = (2, 3) *and* g = 6, *then* $\Phi_{|E|}(S)$ *is a hyperquadric section of the intersection of* $V_5 = G(V, 2) \cap \mathbf{P} \subset \mathbf{P} \simeq \mathbf{P}^6$.

Remark 3. (*i*) The intersection $G(V, r) \cap \mathbf{P} \subset \mathbf{P}$ is not complete but $\sum_g$ is a complete intersection in $G(V, r)$ with respect to the vector bundle $\bigwedge^{r-1} \mathscr{E} \otimes I_{r-1}^\vee$.

(*ii*) If $g = 7$, then there exists a rank 5 vector bundle E with $c_1(E) = 2c_1(L)$ which satisfies the properties *i* and *ii* of *Theorem 3*. The kernel of the natural homomorphism $S^2H^0(S, E) \to H^0(S, S^2E)$ is generated by a nondegenerate symmetric tensor σ. E and σ induce an isomorphism between S and a linear section of the 10-dimensional spinor variety associated to $(H^0(S, E), \sigma)$.

(*iii*) Among other values, $(r, s) = (4, 5)$ is interesting. For every general (polarized) K3 surface (S, L) with $(L^2) = 38$, $\Phi_{|E|}$ is an isomorphism from S onto the intersection $G(V, 4) \cap \mathbf{P} \subset \mathbf{P} \simeq \mathbf{P}^{20}$. It is easily deduced from this fact that the moduli space of polarized K3 surfaces of genus 20 is unirational.

Let $X \subset \mathbf{P}$ be as in *Theorem 2*. By ref. 13, $X \otimes \mathbf{C}$ has a surface section S with Picard number one. The vector bundle E on S in *Theorem 4* extends to a vector bundle $\tilde{E}$ on $X \otimes \mathbf{C}$. This vector bundle $\tilde{E}$ is stable and has property *ii* of *Theorem 3*. By this uniqueness property, $\tilde{E}$ is defined over k. This is a key of the classification over k. The Grassmann embedding of X by $\tilde{E}$ yields *Theorem 2*.

Section 3. Fano 3-Fold of Genus 12 and a Generalized Grassmann Variety

A Fano 3-fold V_{22} of the first species of genus 12 was first found by Iskovskih (14). Unlike other F-manifolds of the first species, V_{22} is not isomorphic to a complete intersection of hypersurfaces in a homogeneous space or to its double cover. But V_{22} has rich geometric structures: It relates with the classical theory of polar polygons (cf. section 57 of ref. 15) and nets of quadrics. Let C be a plane curve of degree d and $F(X, Y, Z) = 0$ be its defining equation. A set of n lines $l_i: f_i(X, Y, Z) = 0$, $1 \leq i \leq n$, is a *polar* n-*side of* C if there exists a set of constants $a_1, \ldots, a_n \in \mathbf{C}$ such that $F = \sum_{i=1}^n a_i f_i^d$.

THEOREM 5. (i) *The variety of polar* 3-*sides of a smooth conic is a smooth quintic del Pezzo* 3-*fold in* $\mathbf{P}^6$.

(ii) *Assume that a quartic curve* C *has no polar 5-sides (see sections 57 and 77 of ref. 15). Then the variety* $P_6(C)$ *of polar 6-sides of* C *is a Gorenstein Fano 3-fold of genus* 12. *If* C *has no complete quadrangles as its polar 6-sides, then* $P_6(C)$ *is smooth and of the first species. The mapping* $C \mapsto P_6(C)$ *gives a birational equivalence between the moduli spaces of quartics* C *and of 3-folds* V_{22}.

Remark 4. If the quartic curve C is a double conic, then the variety of its polar 6-sides is a very special V_{22} with an action of Aut $C \simeq SO(3, \mathbf{C})$. This variety was earlier found by Mukai and Umemura (16) as a smooth equivariant compactification of $SO(3, \mathbf{C})$/(icosahedral group).

In *Section 2*, we embedded V_{22} into a Grassmann variety by a rank 3 vector bundle $\tilde{E}$, applying *Theorem 3* to $(r, s) = (3, 4)$. Applying the theorem to $(r, s) = (2, 6)$, we obtain a rank 2 vector bundle $\tilde{F}$ on V_{22}, too. The pair of two vector bundles $\tilde{E}$ and $\tilde{F}$ leads us to another classification of V_{22}. Instead of $H^0(V_{22}, \tilde{E})$, we consider $V = \mathrm{Hom}(\tilde{E}, \tilde{F})$, which is a vector space of dimension 4. We obtain the natural homomorphism

$$\text{(A)} \qquad V \otimes \tilde{E} \to \tilde{F}.$$

Let $\mathcal{T}$ be the $PGL(V)$-equivariant compactification of the variety of twisted cubics in $\mathbf{P}^*(V)$ constructed in ref. 17. On $\mathcal{T}$, there exist a rank 2 and a rank 3 vector bundles $\mathcal{F}$ and $\mathcal{E}$ and a (universal) exact sequence

$$(\mathcal{A}) \qquad V \otimes \mathcal{E} \to \mathcal{F}.$$

We obtain the unique morphism $\Phi_{\tilde{E},\tilde{F}}: V_{22} \to \mathcal{T}$ with $\Phi^*_{\tilde{E},\tilde{F}}(\mathcal{A}) \simeq (A)$. (This justifies calling $\mathcal{T}$ a generalized Grassmann variety.)

THEOREM 6. *Let* V_{22} *be a Fano 3-fold of the first species of genus* 12 *and let* $\mathcal{T}$ *be as above. Then there exists a net* N *of quadrics in* $\mathbf{P}^*(V)$ *such that* V_{22} *is isomorphic to the closure* $\mathcal{T}_N \subset \mathcal{T}$ *of the variety of twisted cubics defined by three quadratic forms perpendicular to* N. N *is uniquely determined up to* $GL(V)$ *by the isomorphism class of* V_{22}.

Remark 5. The correspondence between N and $\mathcal{T}_N$ in *Theorem 6* is compatible with that in Barth (18): If the net N is of rank 2, then $\mathcal{T}_N$ is singular and is an anticanonical model of a $\mathbf{P}^1$-bundle over $\mathbf{P}^2$ associated to a rank 2 stable vector bundle with $c_1 = 0$ and $c_2 = 4$.

Section 4. Classification by Means of Extremal Rays

Among the simplest examples of Fano manifolds of coindex 3 are the products $\mathbf{P}^1 \times M$ of $\mathbf{P}^1$ and Fano 3-folds M of even index. They are called Fano 4-folds of product type. There are exactly nine deformation types of them corresponding to the nine deformation types of M: $\mathbf{P}^3$, V_7, W, $\mathbf{P}^1 \times \mathbf{P}^1 \times \mathbf{P}^1$, and V_d $(1 \le d \le 5)$, where V_d is a Fano 3-fold of index 2 and $(-K_V)^3 = 8d$ and W is a divisor of bidegree (1, 1) on $\mathbf{P}^2 \times \mathbf{P}^2$ (see refs. 4, 5, 10).

Example 2. The following are Fano manifolds of coindex 3 and of genus g.

1. ($g = 7$): a double cover of $\mathbf{P}^2 \times \mathbf{P}^2$ whose branch locus is a divisor of bidegree (2, 2)
2. ($g = 9$): a divisor of $\mathbf{P}^2 \times \mathbf{P}^3$ of bidegree (1, 2)
3. ($g = 11$): $\mathbf{P}^3 \times \mathbf{P}^3$
4. ($g = 11$): $\mathbf{P}^2 \times Q^3$
5. ($g = 12$): the blow-up of a smooth 4-dimensional quadric $Q^4 \subset \mathbf{P}^5$ along a conic not contained in a plane in Q^4
6. ($g = 13$): the flag variety of $Sp(2)$ [or equivalently, of $SO(5)$]
7. ($g = 14$): the blow-up of $\mathbf{P}^5$ along a line
8. ($g = 16$): the $\mathbf{P}^1$-bundle $\mathbf{P}(\mathcal{O}_Q \oplus \mathcal{O}_Q(1))$ over $Q^3 \subset \mathbf{P}^4$
9. ($g = 21$): the $\mathbf{P}^1$-bundle $\mathbf{P}(\mathcal{O}_\mathbf{P} \oplus \mathcal{O}_\mathbf{P}(2))$ over $\mathbf{P}^3$

For each Fano manifold X in *Example 2*, the fundamental linear system $|H|$ is very ample and the image of $\Phi_{|H|}$ is an F-manifold, which we call the fundamental model of X.

THEOREM 7. *Let* X *be a Fano manifold of coindex* 3, *of dimension* ≥ 4, *and with* $B_2 \ge 2$. *If* X *is not a Fano 4-fold of product type, then* X *is isomorphic to a linear section of the fundamental model of a Fano manifold in* Example 2.

If X is a Fano manifold, then the cone $NE(X)$ of effective (algebraic) 1-cycles on X is spanned by a finite number of extremal rays (see ref. 19). For the proof of *Theorem 7*, the following classification of extremal rays on 4-folds plays an essential role.

THEOREM 8. *Let* R *be an extremal ray of a smooth projective 4-fold. Assume that* R *is of index* ≥ 2; *that is, there exist a divisor* D *and an integer* $r \ge 2$ *with* $(K_X + rD.R) = 0$. *Let* $f = \mathrm{cont}_R: X \to Z$ *be the contraction morphism of* R (see ref. 20).

(i) *If* $\dim Z = 4$, *then* f *is birational and contracts a divisor to a point or to a smooth curve* C. *In the latter case,* Z *is smooth and* f^{-1} *is the blowing up with center* C.

(ii) *If* $\dim Z \le 3$, *then* Z *is smooth and the generic fiber* X_η *of* f *is a Fano manifold of index* ≥ 2 *over the function field* $k(\eta)$ *of* Z.

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